EAGE

CONFERENCE & EXHIBITION

NEAR SURFACE GEOSCIENCE'25

31st European Meeting of Environmental and Engineering Geophysics

6th Conference on Geophysics for Mineral Exploration and Mining

3rd Conference on Hydrogeophysics
Contribution to Exploration and Management of Groundwater, Land-Use and Natural Hazards under a Changing Climate

1st Conference on Geohazard Assessment and Risk Mitigation

1st Conference on UXO & Object Detection

Practical applications of global optimisation techniques in 2-D resistivity inversion

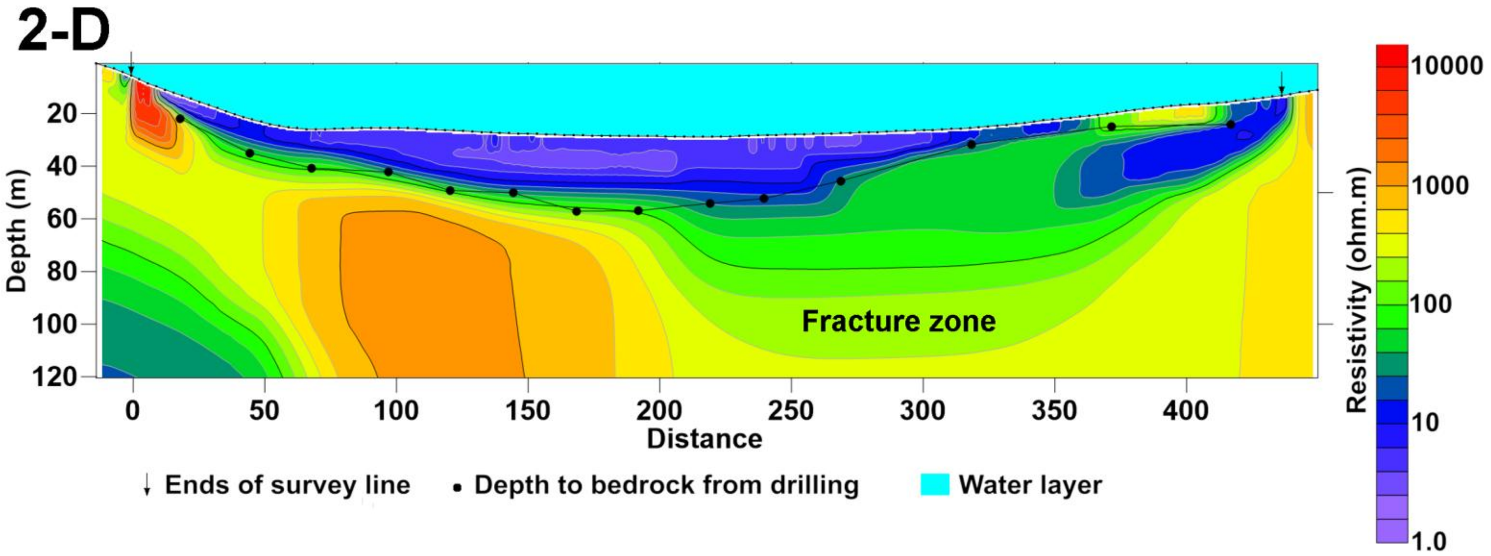
M. H Loke, A. Vinciguerra*, P. B Wilkinson, A. Hojat

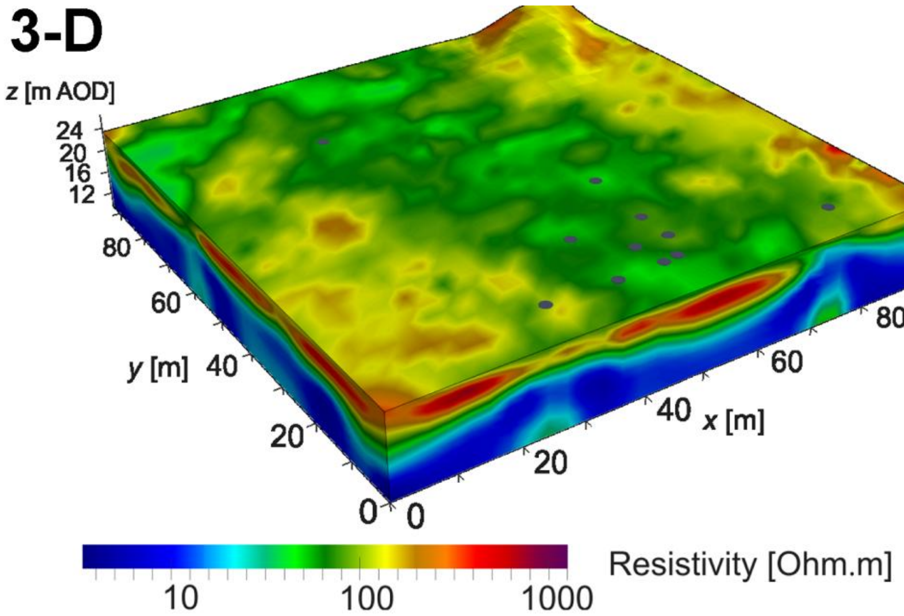
7-11 SEPTEMBER 2025 | NAPLES, ITALY

#NSG2025

2-D and 3-D geoelectrical surveys

- Surveys are usually carried out by small geophysical companies with limited resources
- Field surveys can produce hundreds to thousands of data points.
- Subsurface geology can be very complex, simple 1-D/1.5-D layered models or 2-D/3-D rectangular prisms not sufficiently accurate.
- Inverted model must be able to handle complex geological models.





Nonlinear optimisation methods

Geoelectrical field data are inverted using a nonlinear optimisation method to find the minimum of an objective function *P*(**y**,**q**).

$$P(\mathbf{y},\mathbf{q}) = \psi(\mathbf{y}) + \lambda\phi(\mathbf{q})$$

$P(\mathbf{y},\mathbf{q})$: objective function

$\psi(\mathbf{y})$: data misfit, $\phi(\mathbf{q})$: model roughness

$\mathbf{y}$: data values, $\mathbf{q}$: model parameters

λ: model roughness weight

Nonlinear optimisation methods

Geoelectrical field data are inverted using a nonlinear optimisation method to find the minimum of an objective function $P(\mathbf{y},\mathbf{q})$.

$$P(\mathbf{y},\mathbf{q}) = \psi(\mathbf{y}) + \lambda\phi(\mathbf{q})$$

$P(\mathbf{y},\mathbf{q})$: objective function

$\psi(\mathbf{y})$: data misfit, $\phi(\mathbf{q})$: model roughness

$\mathbf{y}$: data values, $\mathbf{q}$: model parameters

λ: model roughness weight

<u>Local methods</u>:

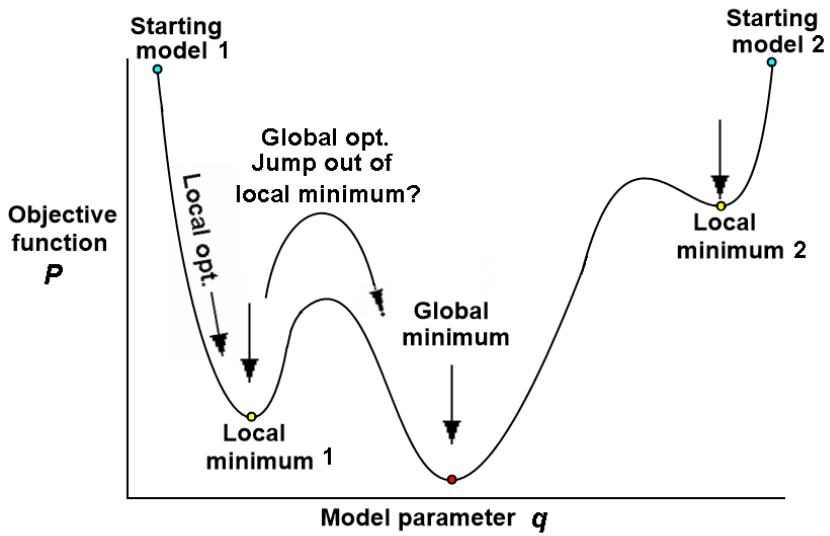
take less time to reach a minimum of the objective function ☺

it might not be the optimal global minimum ☹

<u>Global methods</u>:

theoretically can reach the global minimum ☺

several orders of magnitude slower than local methods ☹



The objective function

Objective function equation $P(\mathbf{y},\mathbf{q})$ for a 2-D model where the L2-norm data and model constraints are used

$$\text{Objective function : } P(\mathbf{y},\mathbf{q}) = \psi(\mathbf{y}) + \lambda\Phi(\mathbf{q})$$

$$\psi(\mathbf{y}) = \sum_{k=1}^{n_a} \left(y_c(k) - y_m(k)\right)^2$$

$$\Phi(\mathbf{q}) = \Phi_x + \Phi_z + \alpha\Phi_r$$

$$\Phi_x = \sum_{j=1}^{n_z} \sum_{i=1}^{n_x-1} \left(q_m(i,j) - q_m(i+1,j)\right)^2$$

$$\Phi_z = \sum_{i=1}^{n_x} \sum_{j=1}^{n_z-1} \left(q_m(i,j) - q_m(i,j+1)\right)^2$$

$$\Phi_r = \sum_{j=1}^{n_z} \sum_{i=1}^{n_x} \left(q_m(i,j) - q_r\right)^2$$

y_c, y_m = log of calculated and measured apparent resistivity, n_a = number of data points

q_m, q_r = log of model and reference resistivity, n_x, n_z = number of model cells in x, z directions

α = weight of reference model constraint, λ = model roughness regularisation parameter

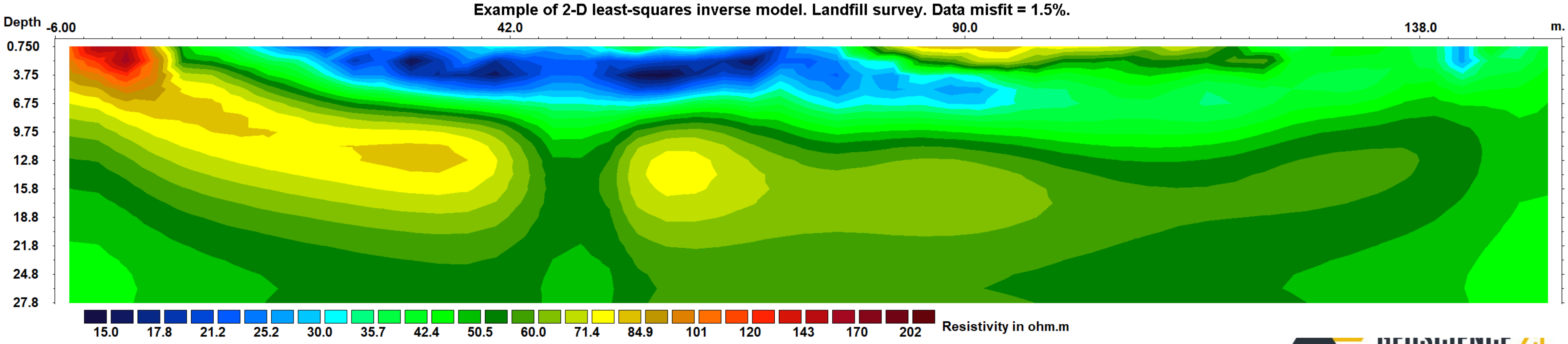
Local optimisation method

The **least-squares** method solves the equation below to determine the change in the model **Δq** that reduces the data misfit $\boldsymbol{y}$. The inversion starts with a large value for the regularisation parameter λ that is slowly reduced after each iteration.

$$(\boldsymbol{J}^T\boldsymbol{J} + \lambda \boldsymbol{F}_R)\Delta \boldsymbol{q}_k = \boldsymbol{J}^T\boldsymbol{y} - \lambda(\boldsymbol{F}_R + \alpha \boldsymbol{I})(\boldsymbol{q}_k - \boldsymbol{q}_R)$$

$\boldsymbol{J}$: Jacobian matrix, $\boldsymbol{q}_k$: model resistivity, $\boldsymbol{q}_R$: reference model, $\boldsymbol{\lambda}$: regularization parameter,

$\boldsymbol{F}_R$: roughness filter matrix, $\boldsymbol{y}$: data misfit, $\Delta\boldsymbol{q}_k$: change in model

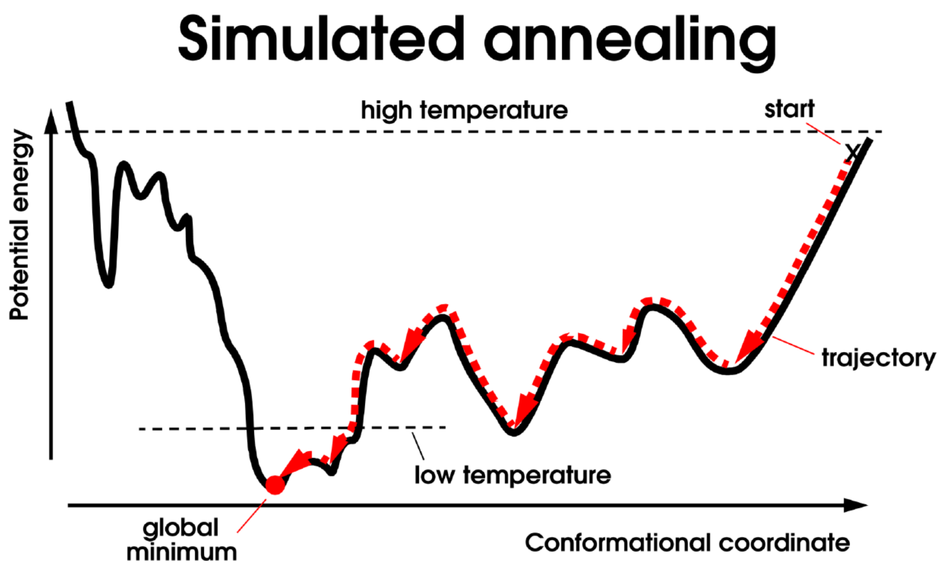


GEOSCIENCE 23

Global optimisation methods

Global optimisation methods include simulated annealing, PSO, Bayesian, genetic algorithms etc.

The **simulated annealing** method attempts to find the global minimum by starting from an initial model and searches in the region around the model to find a lower objective function point (Kirkpatrick et al., 1983, Press et al., 2007).



Simulated Annealing basic step

- Apply random perturbation $\boldsymbol{\Delta q_k}$ to current model $\boldsymbol{q_k}$
- Compute the difference $\Delta P = P(\boldsymbol{y}, \boldsymbol{q_k} + \Delta \boldsymbol{q_k}) - P(\boldsymbol{y}, \boldsymbol{q_k})$
- If $\Delta P < 0$, accept new model: $\boldsymbol{q_k} \rightarrow \boldsymbol{q} + \Delta \boldsymbol{q_k}$
- If $\Delta P > 0$, accept new model with probability $p = e^{-\frac{\Delta P}{T}}$

Source : Peter Güntert, 2013. Strukturbestimmung mit NMR Spektroskopie

Inversion strategy used

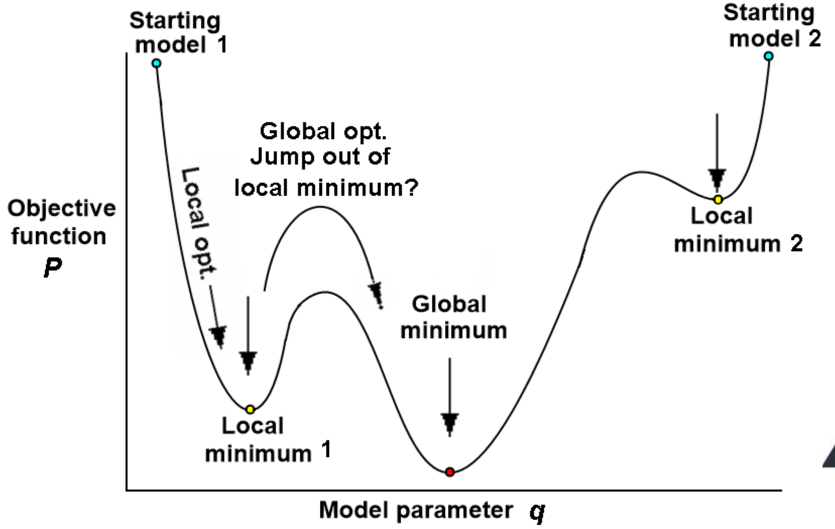
The **least-squares** method converges rapidly, but it might converge to a non-optimal local minimum.
The **simulated annealing** method can find the global minimum but requires thousands of times more trial models than the least-squares method.

Hybrid approach :

- First use the least-squares method to find a model which should be nearer to the global minimum than the starting model.
- Then use the simulated annealing method to jump out of the local minimum to a point closer to the global minimum.

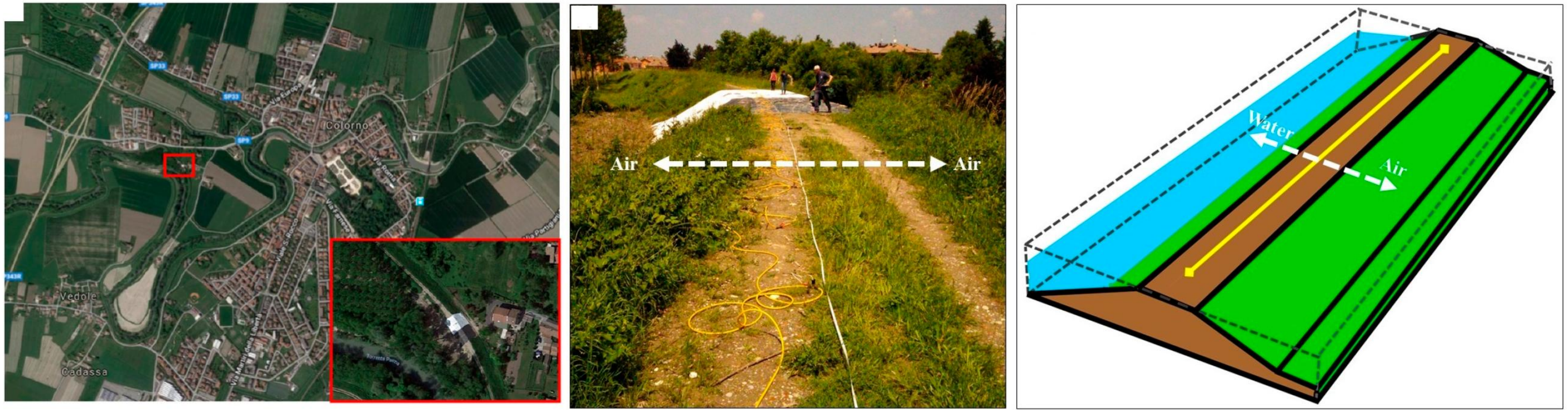
$$P(\mathbf{y},\mathbf{q}) = \psi(\mathbf{y}) + \lambda\phi(\mathbf{q})$$

$P(\mathbf{y},\mathbf{q})$: objective function
$\psi(\mathbf{y})$: data misfit, $\phi(\mathbf{q})$: model roughness
$\mathbf{y}$: data values, $\mathbf{q}$: model parameters
λ: model roughness weight



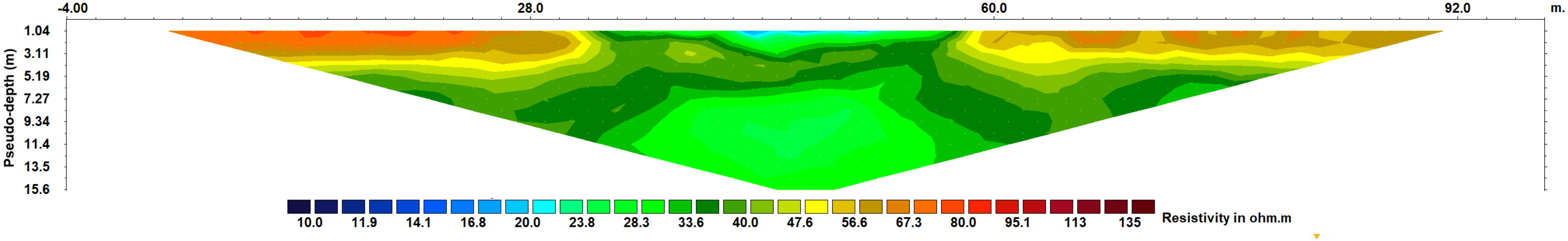
Test data set

A 2-D survey was carried out on part of the Parma river levee near Colorno in central Italy. Corrections were made for changes in the elevation of the levee and the river water for the apparent resistivity data.



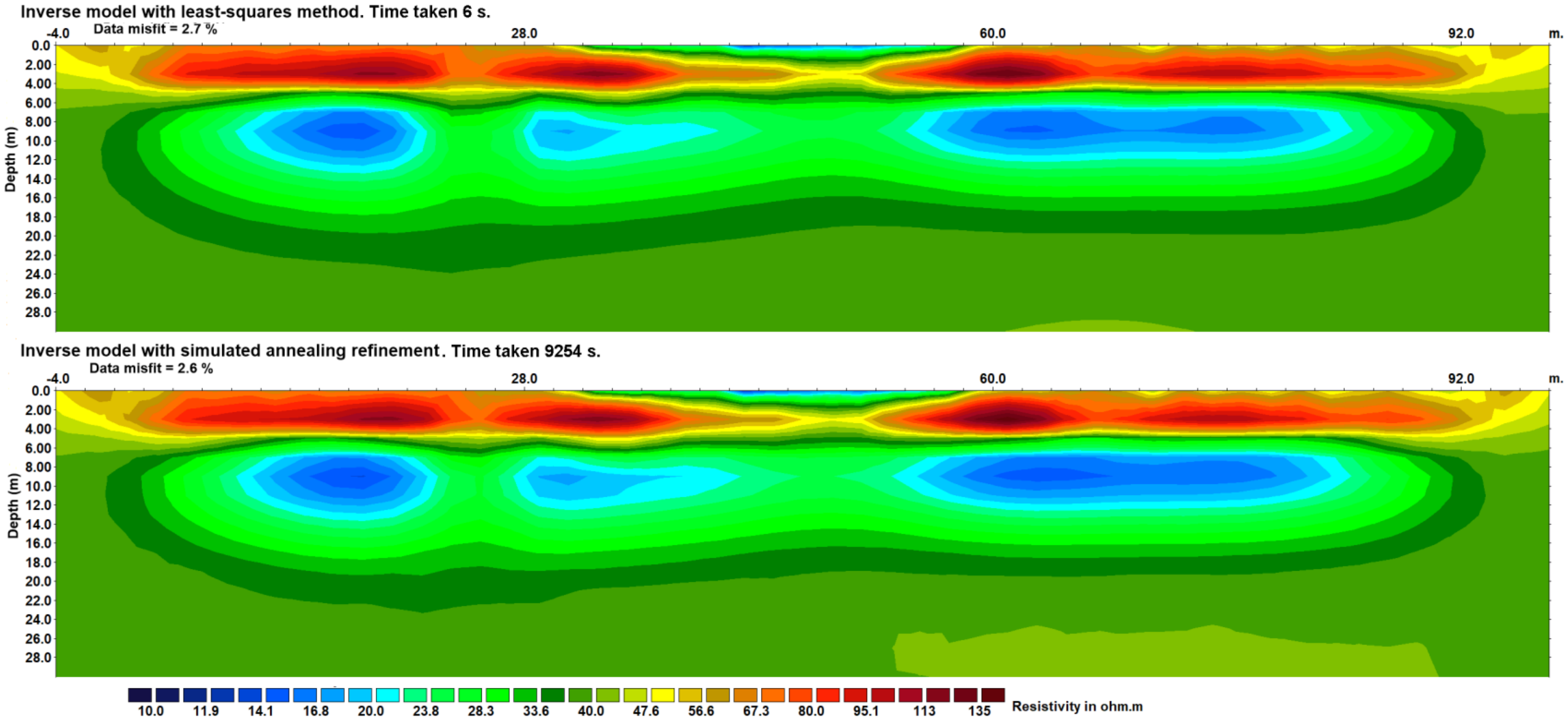
Figures taken from Hojat 2024

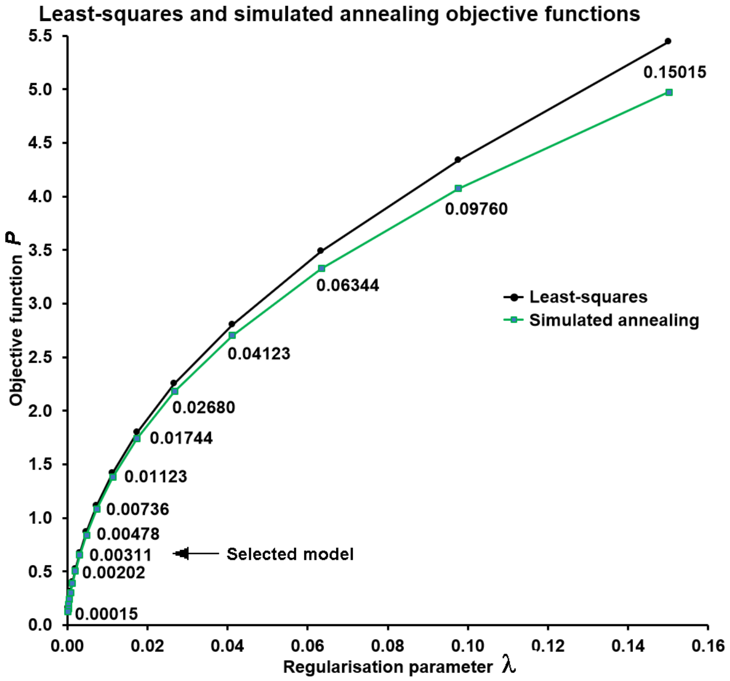
Measured apparent resistivity pseudosection



Inversion results

- Small differences between the simulated annealing and the least-squares models
- The simulated annealing method gives a lower objective function P, the difference decreases with λ.
- Time taken by the simulated annealing method is 1500 times longer.



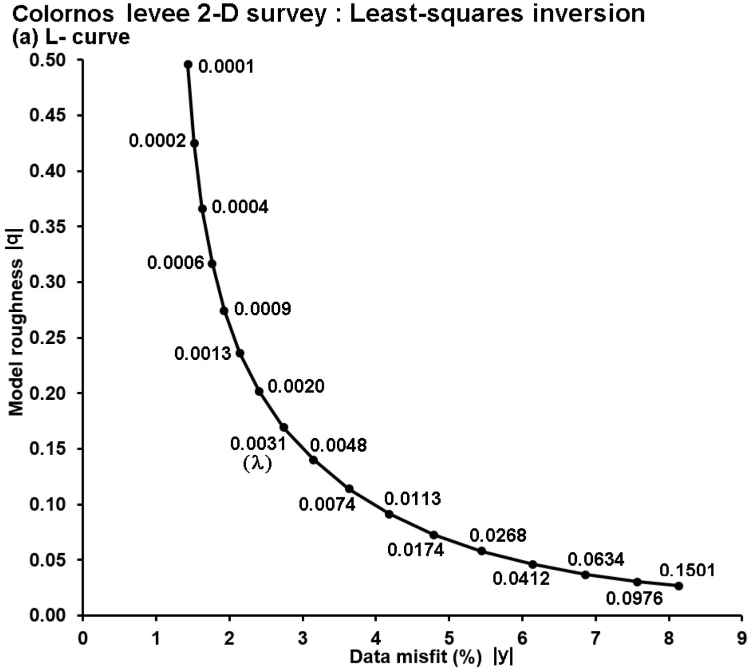


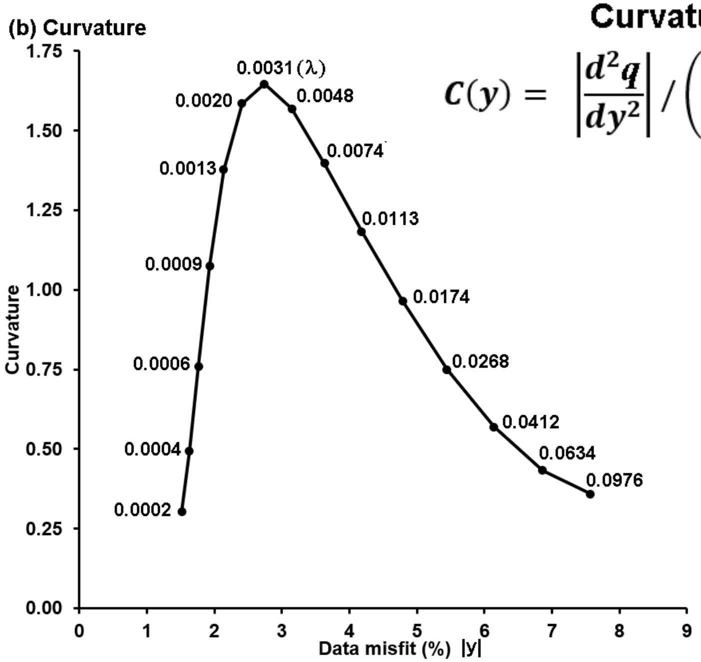
How to determine the optimum value of λ?

$$P(\mathbf{y},\mathbf{q}) = \psi(\mathbf{y}) + \lambda\phi(\mathbf{q}) \qquad P(\mathbf{y},\mathbf{q})\text{: objective function}$$

The L-curve method tries to find the value of λ that balances **y** and **q**.

In this example the L-curve is very smooth so there is a clear maximum in the curvature curve. The curvature is calculated using the second order differential of **q** versus **y**.





Curvature

$$C(y) = \left|\frac{d^2q}{dy^2}\right| \Big/ \left(1 + \left|\frac{dq}{dy}\right|^2\right)^{3/2}$$

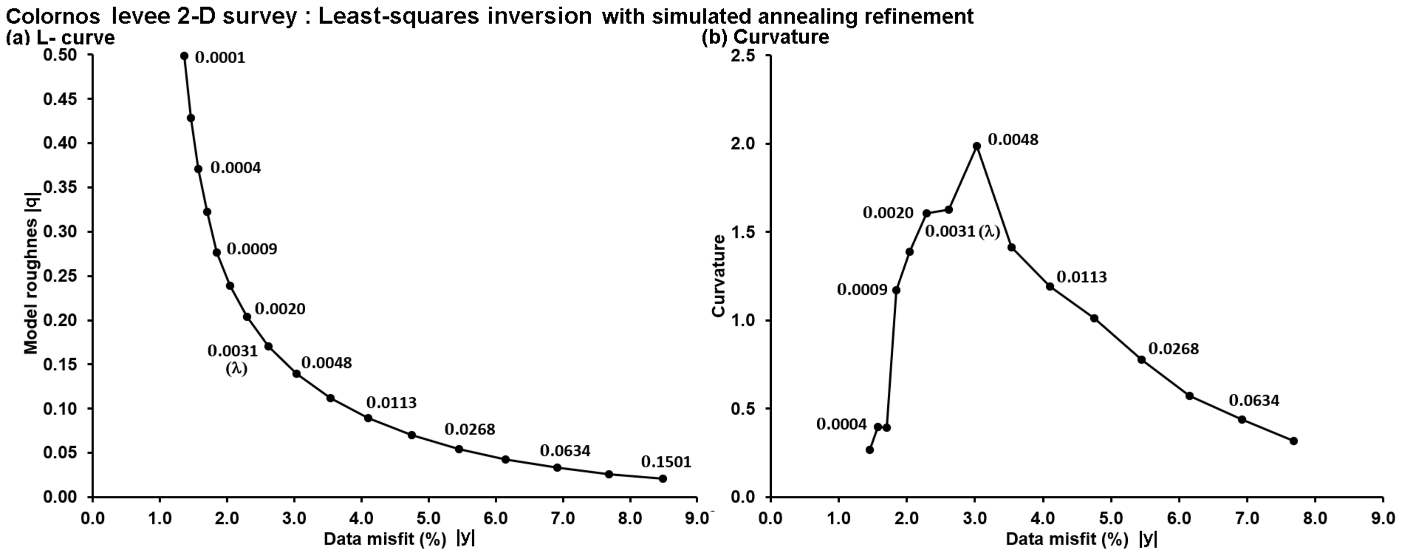
Problems with the L-curve method

- The L-curve use the 2nd order derivative of the model roughness wrt the data misfit to calculate the curvature :

$$C(y) = \left|\frac{d^2q}{dy^2}\right| \Big/ \left(1 + \left|\frac{dq}{dy}\right|^2\right)^{3/2}$$

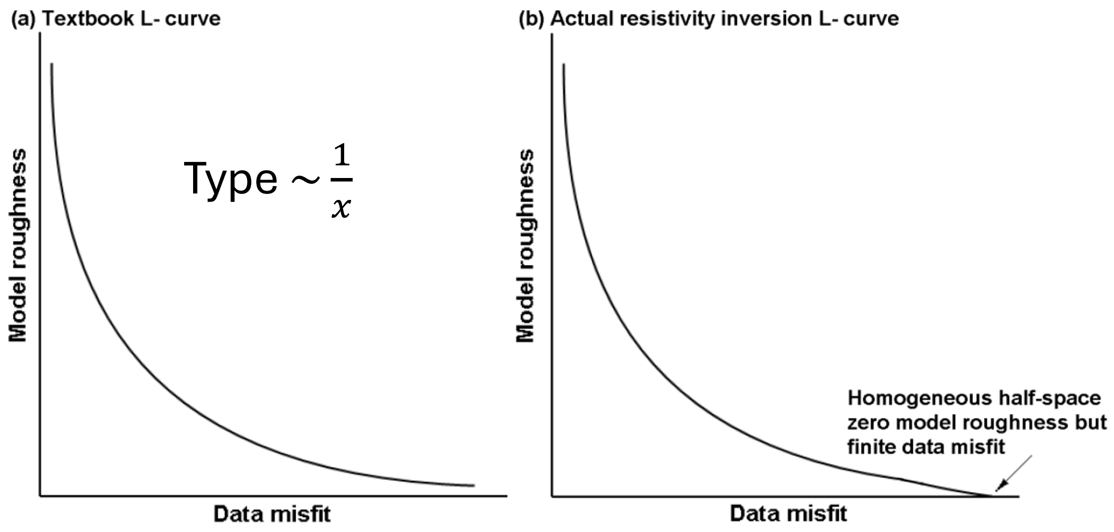
- The L-curve for the inverse model with the simulated annealing refinement shows a less smooth curvature curve with a slight shift in the maximum.

Problem : The derivative is calculated by differences $dq/dy = (q_2\text{-}q_1)/(y_2\text{-}y_1)$.
Small changes in the curve cause large changes in d^2q/dy^2 and the curvature.



A method to check noisy L-curves

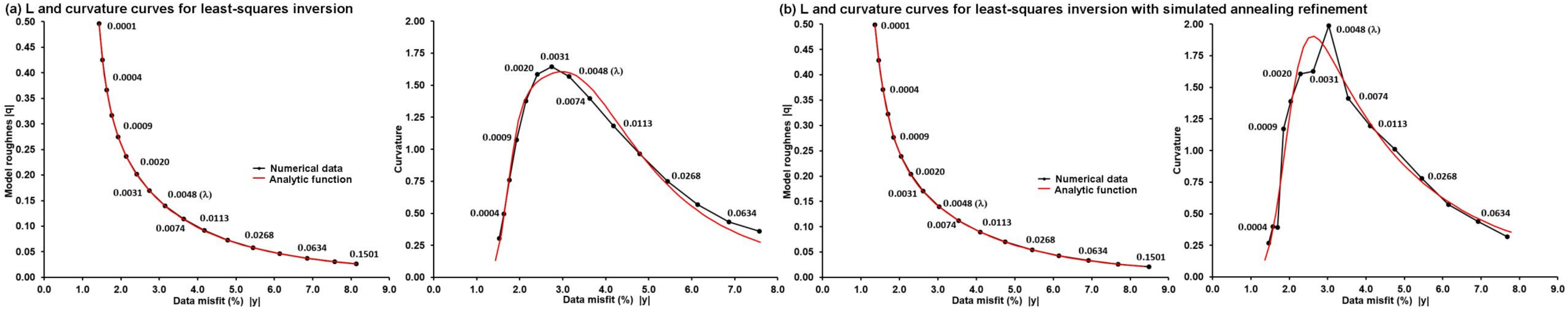
- A homogeneous half-space will have zero model roughness but a finite data misfit, so a real resistivity inversion L-curve has a finite right end.
- One function tested has a linear component and a (1/x) Taylor series.



$$y = \overbrace{mx + c}^{} + \overbrace{a_1(\frac{b_1}{x})^f + a_2(\frac{b_2}{x})^{f/2} + a_3(\frac{b_3}{x})^{f/3} + a_4(\frac{b_4}{x})^{f/4} + \ldots}^{}$$

For four terms in the series, the equation has 11 variables which are determined using a damped least-squares optimisation method.

Analytic curves with the Colornos L-curves

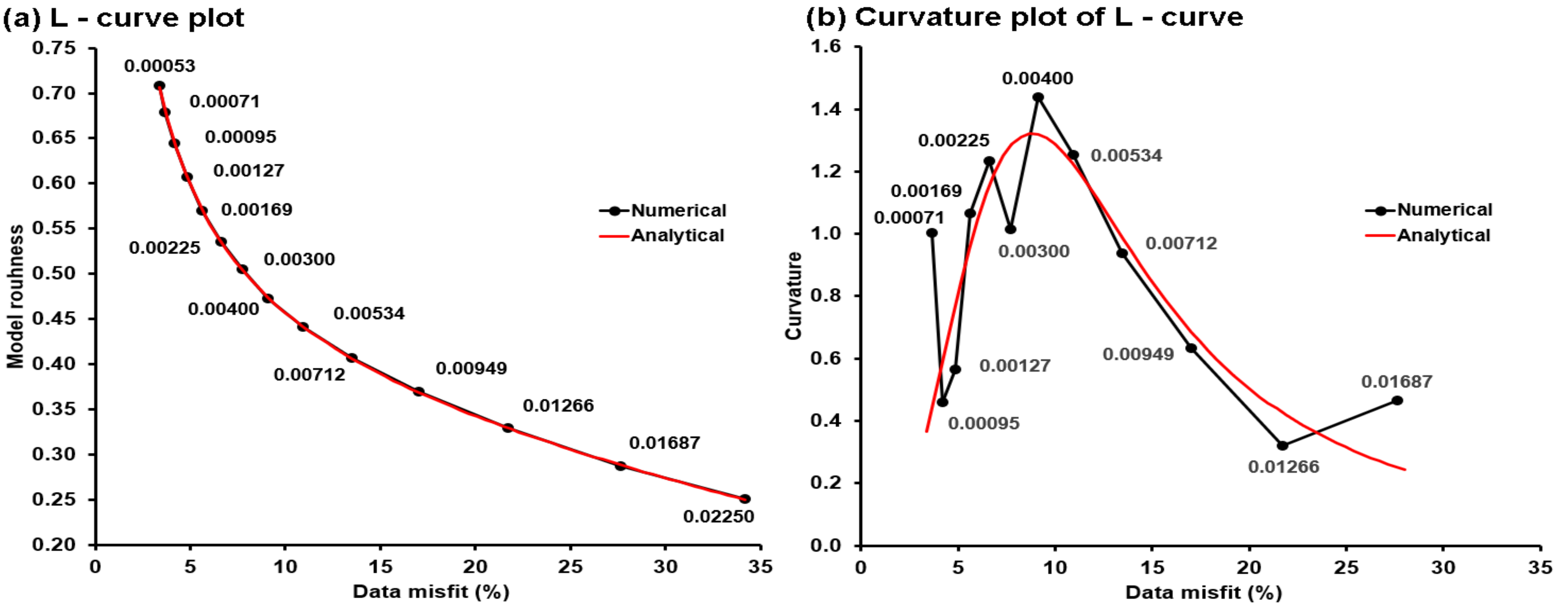


There is a good fit with the numerical L-curves using a 4th order function.

Very small shifts in the L-curve points for the results with simulated annealing refinement between L-curve regularisation parameter (λ) values of 0.0020 and 0.0074 causes a slight distortion in the curvature curve near the maximum.

The analytic curvature curves for both cases show that the optimum λ value is between 0.0031 and 0.0048.

Example of noisy L-curve



Example of noisy numerical curvature plot with more than one maximum from a deep 3-D survey using 100m dipoles. The analytical curve suggests the correct maximum is near the regularisation parameter of 0.004. Although there is a good fit between the numerical and analytical L-curve plots, small deviations from a smooth curve causes large changes in the curvature plot.

Conclusions

- The smoothness-constrained least squares method converges rapidly to an acceptable model within seconds for 2-D survey data sets.

- The use of the simulated annealing method to refine the least-squares model reduces the objective function at the cost of calculation time that is 1500 times longer.

- The optimum regularisation parameter λ can be estimated using the L-curve method. A best fitting analytic function helps to estimate the maximum of the curvature curve for noisy L-curves.

- Research is being carried out using other global optimisation methods such as PSO and MCMC, and an algorithm that uses the global optimisation method alone with a homogeneous half-space starting model.

Thank you for your attention!

Authors :
M.H.Loke - Geotomo Software Sdn Bhd.
A. Vinciguerra - University of Strasbourg.
P. B. Wilkinson - British Geological Survey.
A. Hojat - Politecnico di Milano.