Determination of model reliability in 3-D resistivity and I.P. inversion

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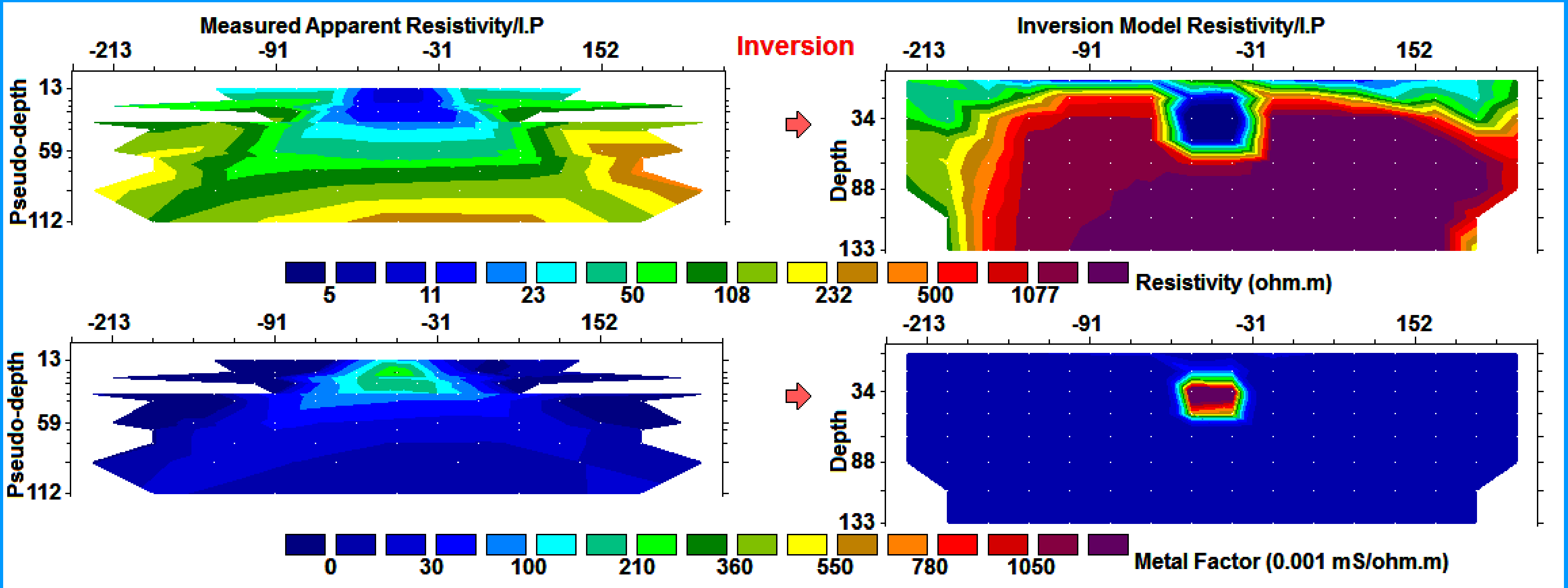
Outline

1. Data inversion and models
2. Sensitivity approach
3. DOI/VOI approach
4. Model resolution approach
5. Examples

From data to models

Data – What we measure in a survey. Apparent resistivity/I.P.

Model – Information about the subsurface. True resistivity/I.P. :- Geology?



Least-squares inversion method

The smoothness-constrained least-squares method is commonly used for large scale problems. It attempts to minimise a combination of the data misfit (g), model roughness (λ) and deviation from a reference model (μ).

$$\left(\mathbf{J}^{\mathbf{T}}\mathbf{J} + \lambda\left(\mathbf{F} + \mu\mathbf{I}\right)\right)\Delta\mathbf{q}_{\mathbf{k}} = \mathbf{J}^{\mathbf{T}}\mathbf{g} - \lambda\left[\mathbf{F}\mathbf{q}_{\mathbf{k}} + \mu\left(\mathbf{q}_{\mathbf{k}} - \mathbf{q}_{\mathbf{m}}\right)\right]$$

F = roughness filters to stabilize inversion

$\boldsymbol{\lambda}$ = roughness filter damping factor, weight for model roughness

$\mathbf{q_k}$ = current inversion model, $\Delta\mathbf{q_k}$ = change in model resistivity/I.P.

$\mathbf{q_m}$ = reference model, $\boldsymbol{\mu}$ = reference model damping factor

g = difference between calculated and measured values, data misfit

J = Jacobian matrix of partial derivatives.

Sensitivity approach

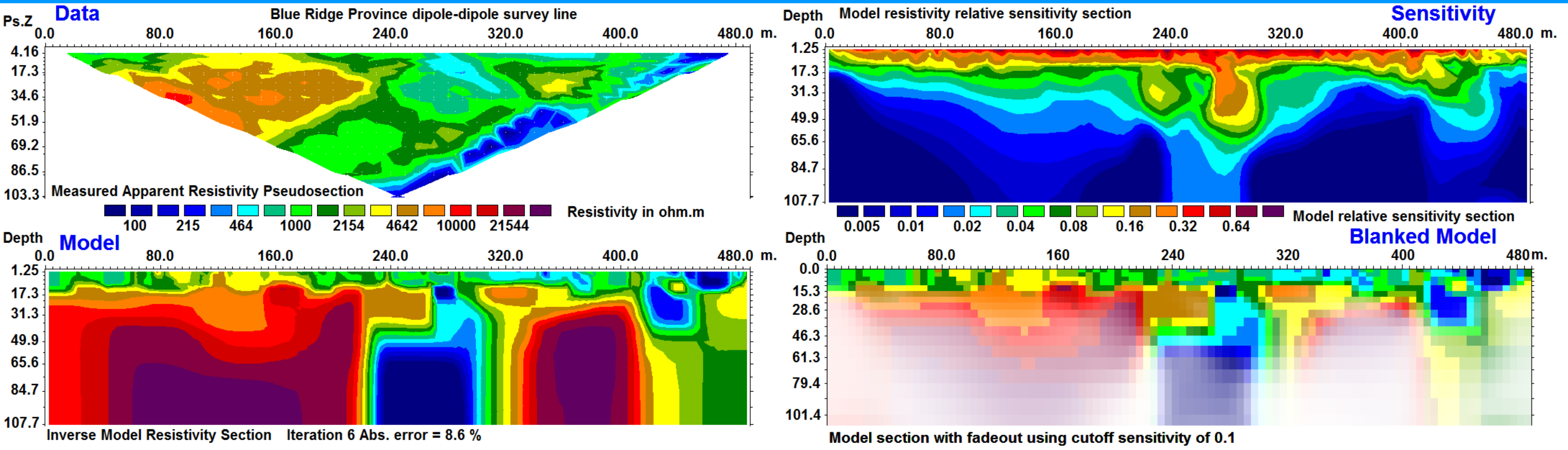
Simplest method. The Jacobian matrix **J** values give the sensitivity of the measured apparent resistivity r_i to the subsurface resistivity model cell q_j.

$$J_{ij} = \frac{\partial r_i}{\partial q_j}, \quad s_j = \sum_{i=1}^{i=m} \left| J_{ij} \right|, \quad S_j = \frac{s_j}{s_{MAX}}$$

For each model cell q_j, calculate the sum of the absolute sensitivity values s_j. This gives an idea of the model cells that affect the measurements. As the sensitivity values can have a wide range, we normalise it by dividing with the maximum value to give a range of 0.0 to 1.0.

2-D survey example – use of sensitivity

We use a 2-D example of a dipole-dipole survey in a hard-rock environment. The sensitivity decreases with depth. Areas with high resistivity generally have lower sensitivity, possibly due to less current penetration. Advantage : Easy to calculate. Disadvantages : Does not take into account overlapping data coverage, arbitrary cutoff value.



DOI/VOI approach

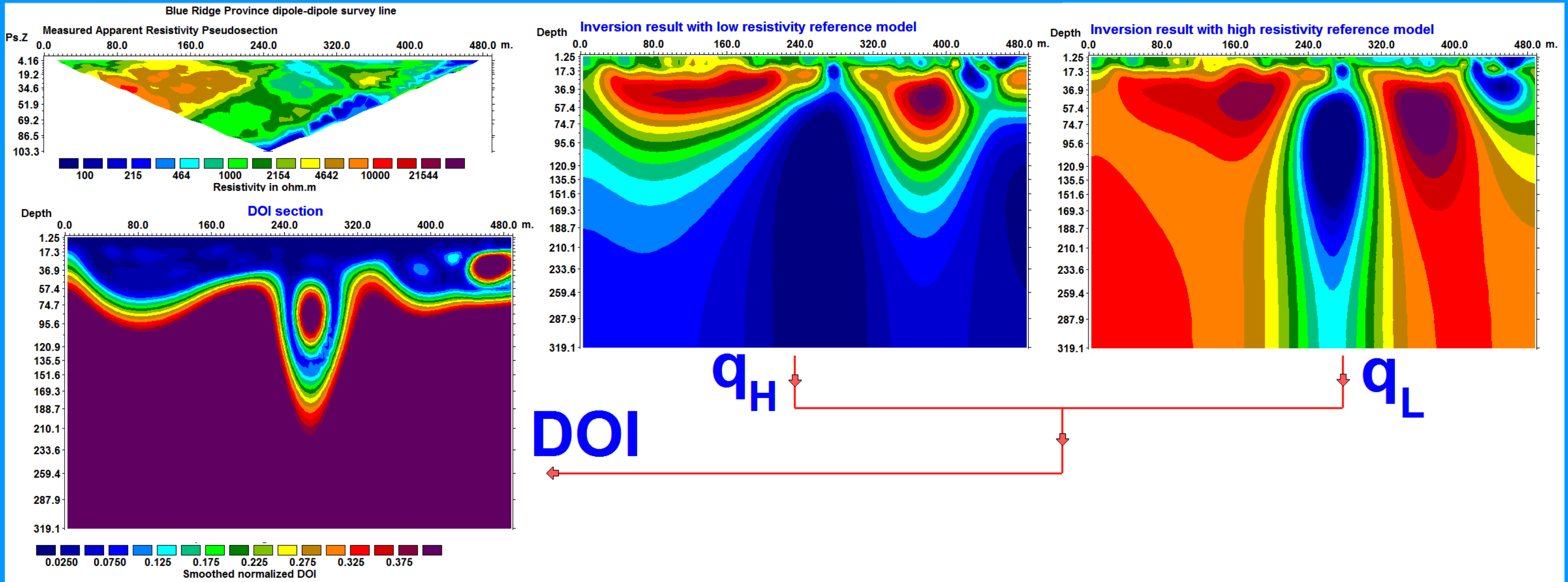
This method carries out two separate inversions using different reference models, $\mathbf{q_m}$, such as 0.1 and 10 times the average apparent resistivity value. In areas with good data coverage, both models ($\mathbf{q_H}$ and $\mathbf{q_L}$) should give similar model resistivity values :- DOI value will approach 0.0. In areas with poor information, the models will tend to the reference models used :- DOI value will approach 1.0.

$$\left(\mathbf{J}^{\mathbf{T}}\mathbf{J} + \lambda\left(\mathbf{F} + \mu\mathbf{I}\right)\right)\Delta\mathbf{q_k} = \mathbf{J}^{\mathbf{T}}\mathbf{g} - \lambda[\mathbf{F}\mathbf{q_k} + \mu\left(\mathbf{q_k} - \mathbf{q_m}\right)]$$

$$DOI(x,z) = \frac{q_H(x,z) - q_L(x,z)}{q_{mH} - q_{mL}}$$

2-D DOI example

The model section is extended downwards to several times the maximum depth of investigation to ensure it includes areas with no sensitivity, i.e. DOI of 1.0. There is usually a sharp boundary between areas of low and high DOI. On undesirable feature is shallow localized regions of high DOI, probably artefacts due to the local optimisation method used. A cutoff value of about 0.1 is commonly used.



Model resolution approach

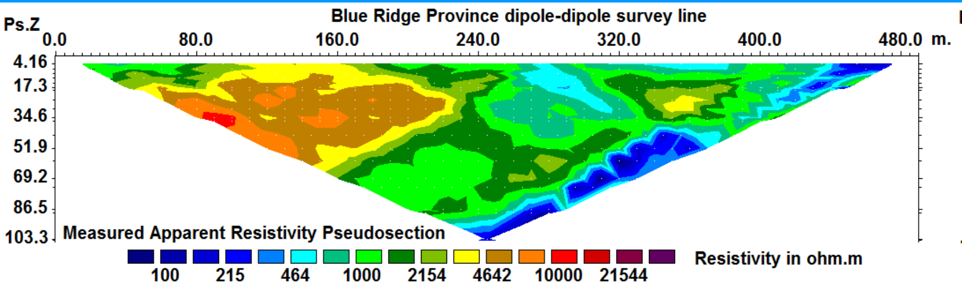
The model resolution matrix **R** is closely related to the least-squares optimisation method. It is given by

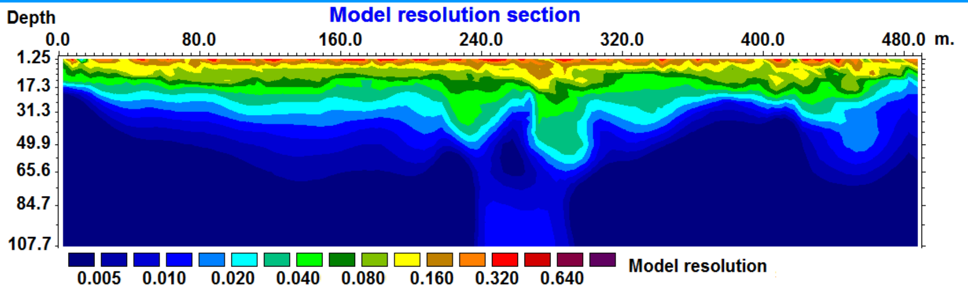
$$\mathbf{R} = \left[\mathbf{J}^{\mathrm{T}}\mathbf{J} + \lambda\mathbf{F}\right]^{-1}\mathbf{J}^{\mathrm{T}}\mathbf{J}$$

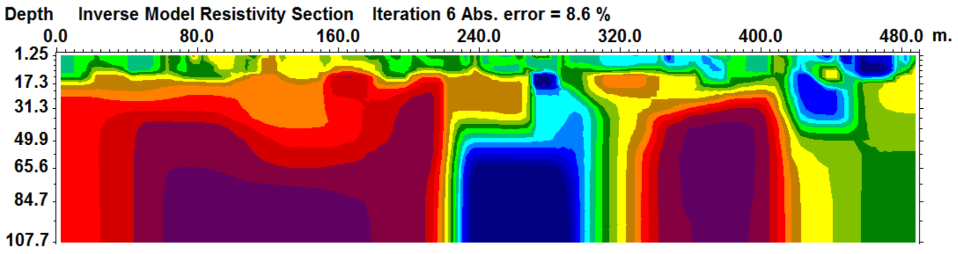
The main diagonal elements of **R** give the resolution of the model cells. It has a value approaching 1.0 in regions that are well constrained by the data, and 0.0 where there is no information. It involves a matrix inversion that requires m^3 calculations. For large 3-D models, the number of model cells m can exceed 10,000.

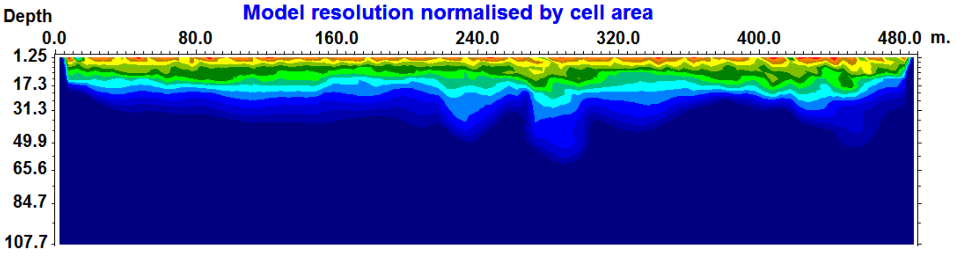
2-D model resolution example

The model resolution generally decreases with depth, and shows fewer artefacts compared to the DOI section. The model resolution also depends on the size of the model cell, which causes a slight increase near the bottom due to thicker model layers. This effect is reduced by normalising the resolution values by the size of the model cell. A value of 0.05 has been used as the model resolution cutoff value, but this might be a little arbitrary.









Model resolution index

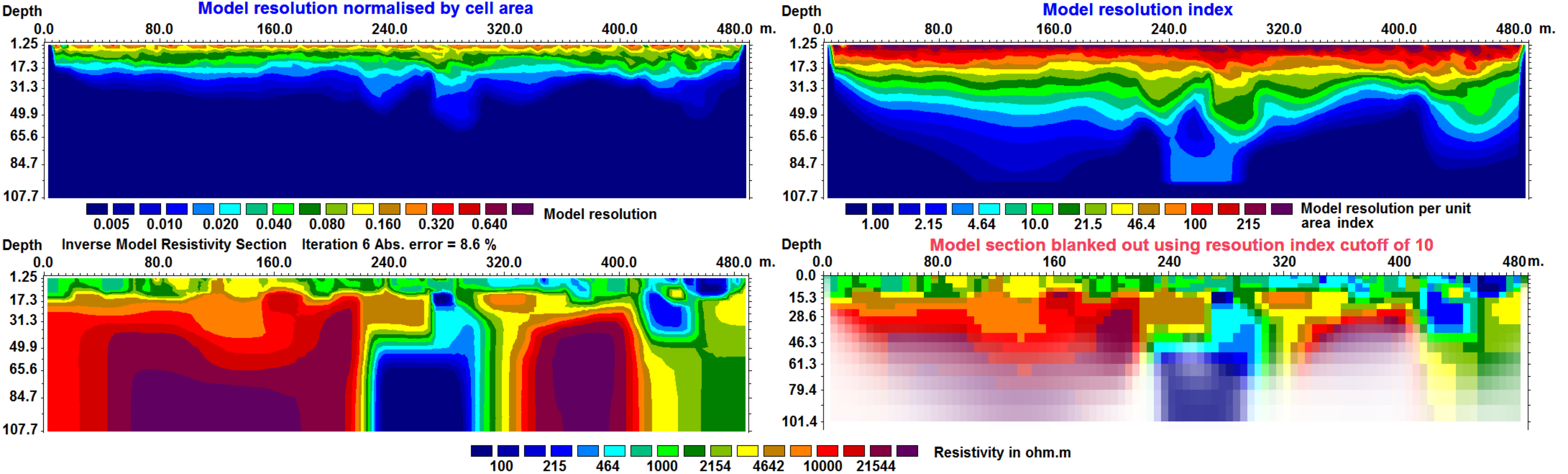
The resolution values also depend on how finely the subsurface is subdivided into model cells. A discretization with smaller model cells will reduce the resolution value for a cell at the same location. The sum of the elements in a column of the **R** matrix is equals to 1.0. The average value of the column elements is *1.0/m*. To compensate for the effect of the model discretisation, the following index value R_c is used.

$$R_c(i,i) = m * R(i,i)$$

This gives the ratio of the resolution value to the case with random noise.

Model resolution index example

The model resolution index is a scaled version of the resolution section which takes into account the number of model cells. As an example, choosing a value of 10 for the index means selecting cells that have 10 times the resolution compared to random noise.



3-D model reliability calculations

The sensitivity, DOI (VOI) and model resolution methods can be used in the same way for 3-D models. The main problem is one of scale as 3-D problems are one to three orders of magnitude larger than 2-D.

Sensitivity, VOI – Can be used for any problem where it is possible to carry out an inversion. Main problem with sensitivity is in determining a usable cutoff value. VOI usually has sharp transitions, cutoff ~ 0.1.

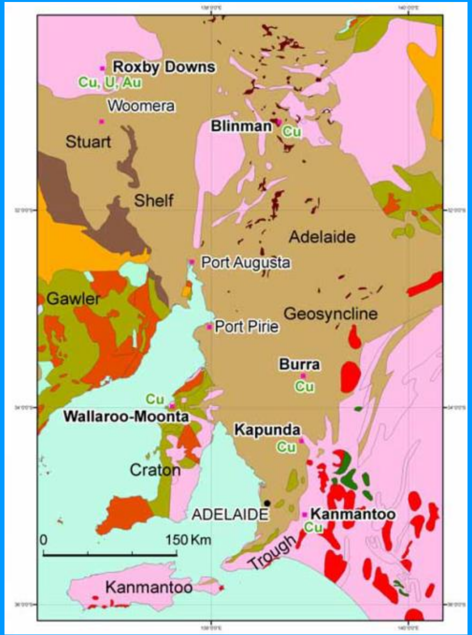
Resolution – Computational limits. n = number of data points, m = number of model cells. Range from 1,000 to 100,000 in 3-D problems.

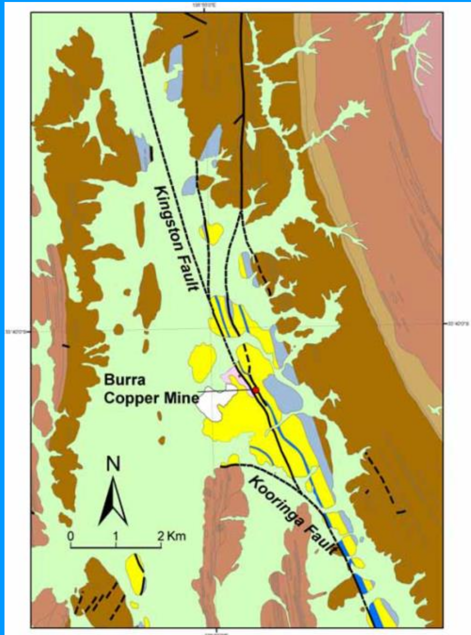
Memory requirements ~ nm and m^2. *32 to 128 GB RAM.*

Calculation time ~ nm^2 and m^3. *Many CPU cores and hours.*

3-D example : Burra copper deposit, SA

It is one of the oldest and largest copper mines in Australia, with active mining in the 1848-1877 and 1971-1981 periods. Recently there has been some interest to renew mining in this area. The main lodes occur near the north-south Kingston Fault.

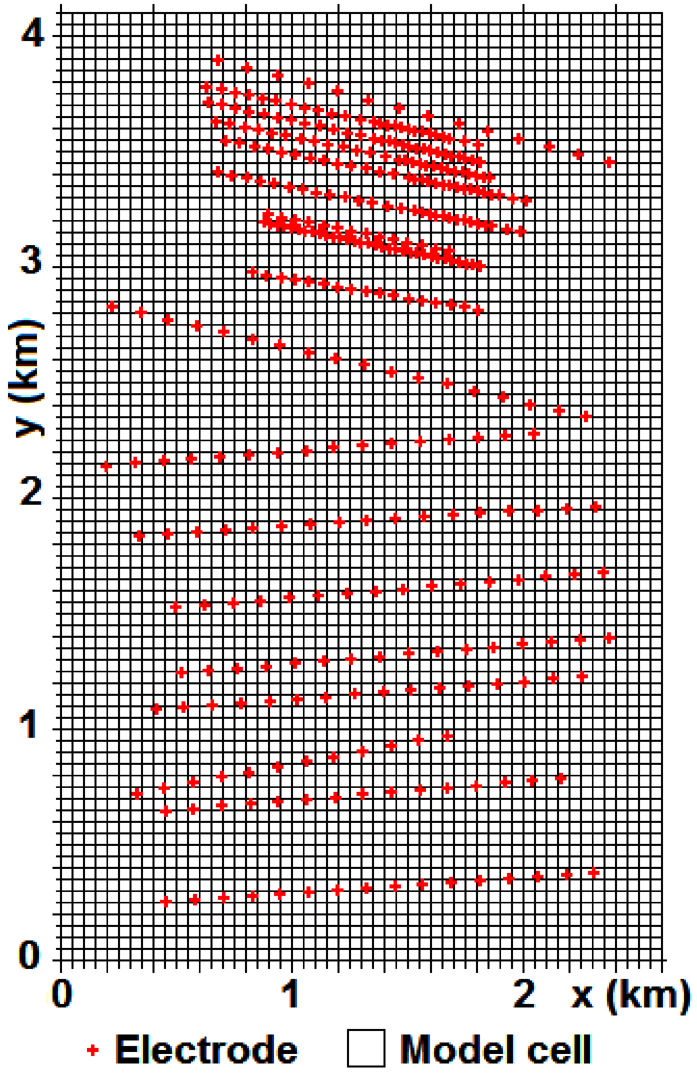




Burra copper deposit - surveys

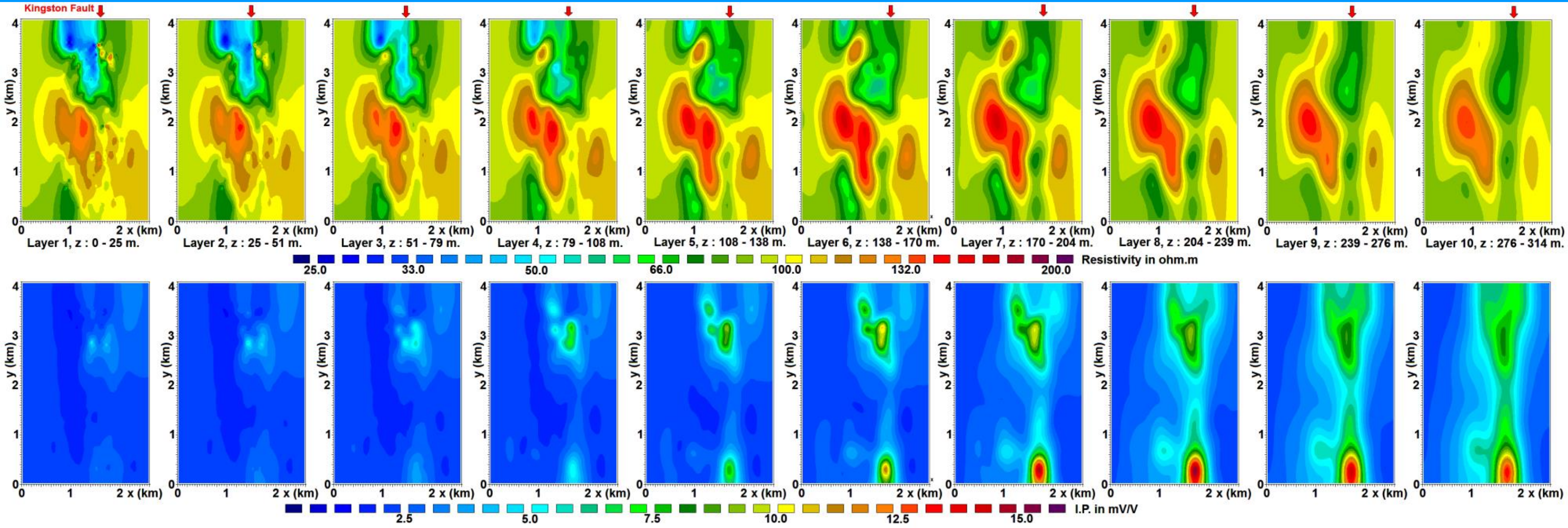
In the 1960's, a number of I.P. surveys along 2-D lines were conducted. A re-interpretation of the data was carried using modern 3-D inversion methods.

There are 777 data points but the model has 58240 models cell, so the inversion problem is highly under-determined.



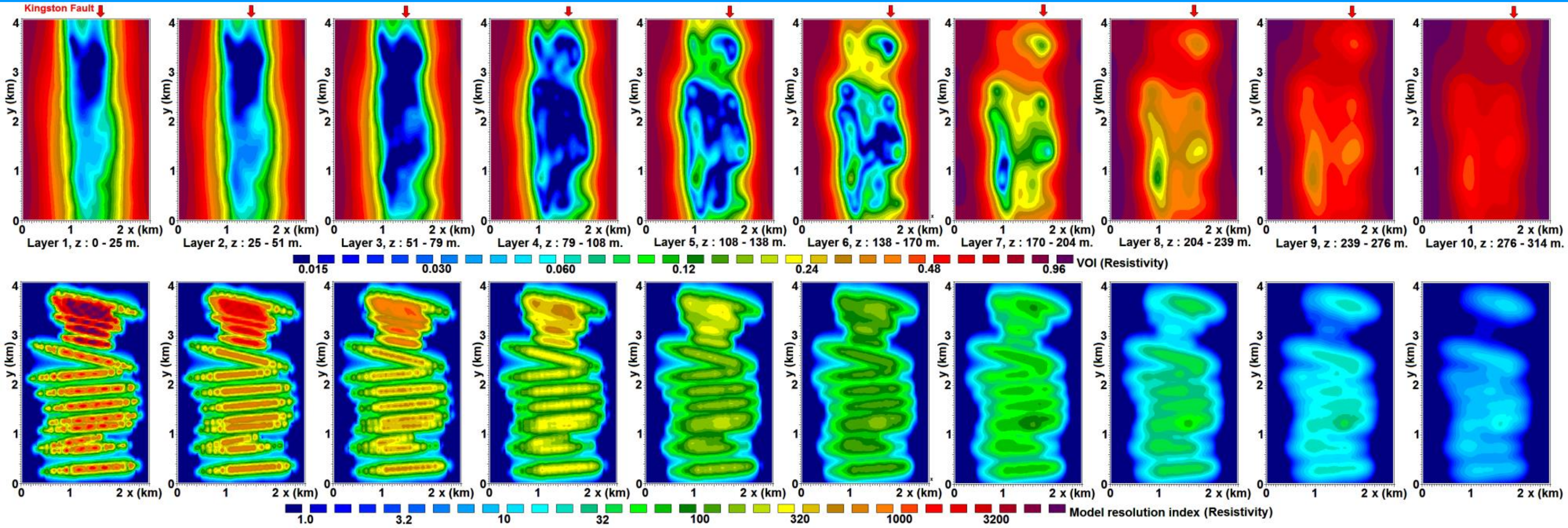
Burra copper deposit – resistivity and I.P. models

The resistivity (top) and I.P. (bottom) models are shown in the form of x-y (EW-NS) slices at different depths from the surface, starting from the left. There are generally lower resistivity values along the approximately north-south Kingston fault, and high I.P. values at two places. The northern I.P. anomaly corresponds to the Eagle deposit prospect.



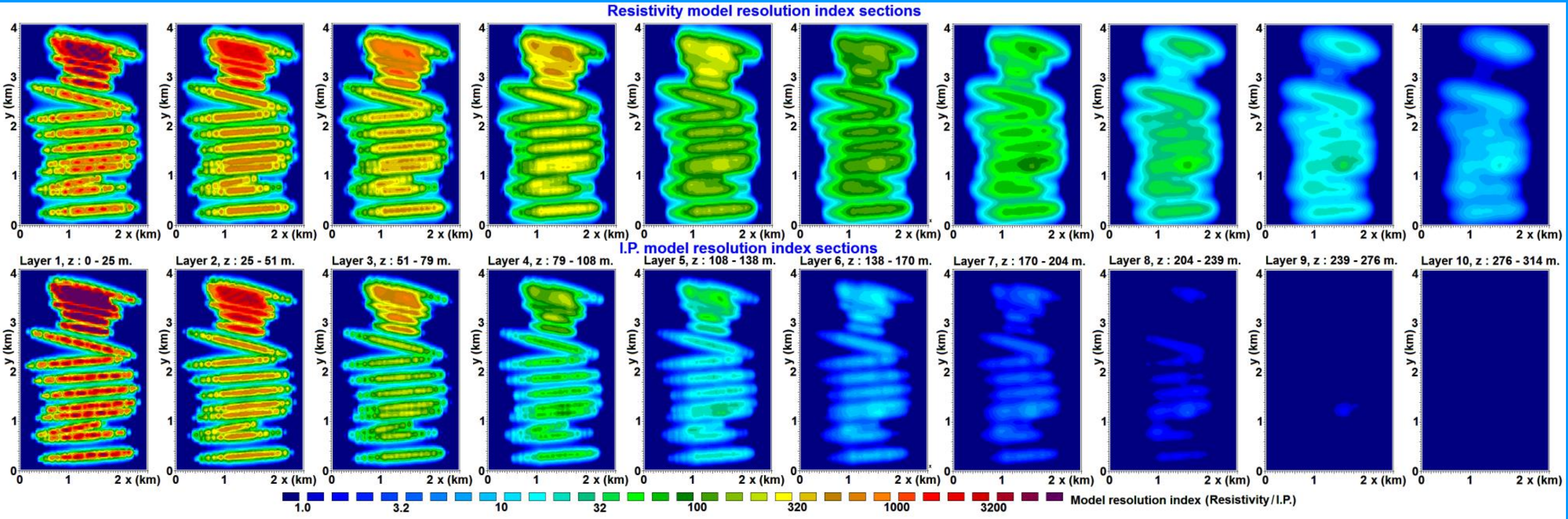
Burra copper deposit – VOI and model resolution

The VOI plot suggests the maximum depth of investigation is about 140-170m. The resolution index plot puts the limit at about 170-200 m. Both plots indicate the southern edge where the second I.P. anomaly is located has very little information. Both plots indicate the topmost part of the northern Eagle prospect I.P. anomaly has reasonable data coverage.



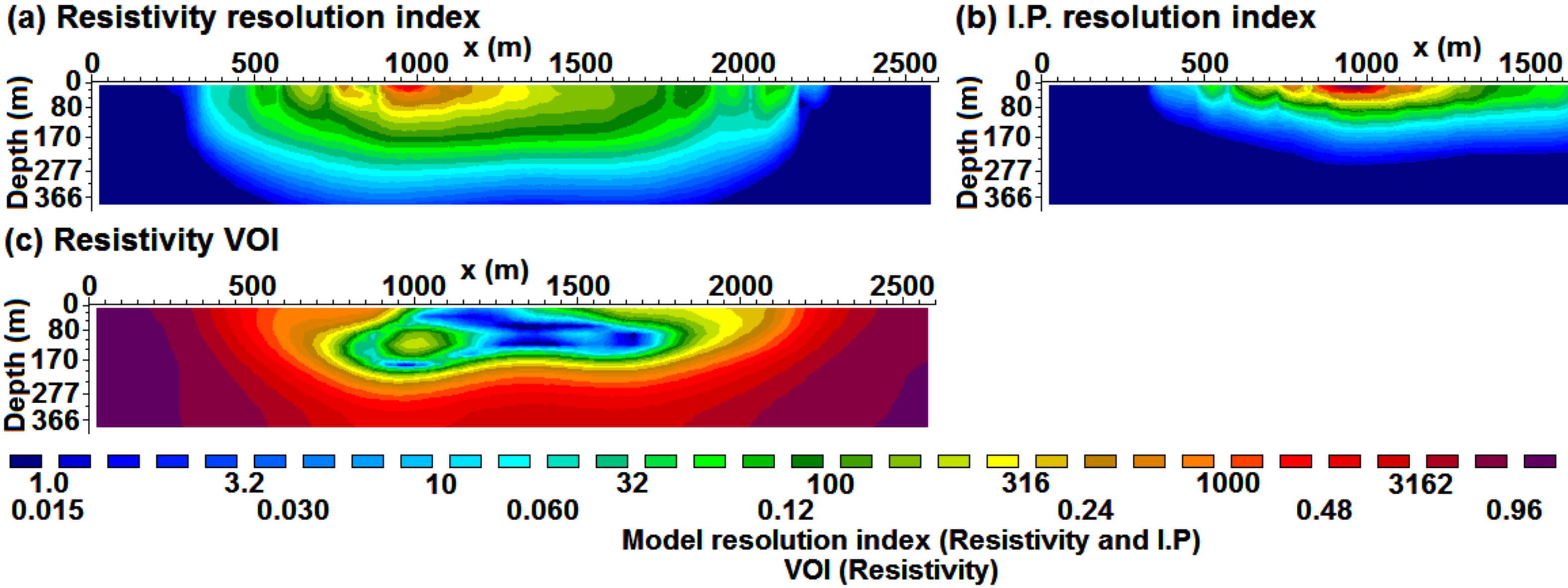
Burra copper deposit – I.P. model resolution

It is generally assumed the I.P. resolution is similar to the resistivity resolution. The plot of the resolution values calculated from the I.P. (bottom) data suggests the maximum depth of investigation is less than for resistivity (top) anomalies, at least for this data set.



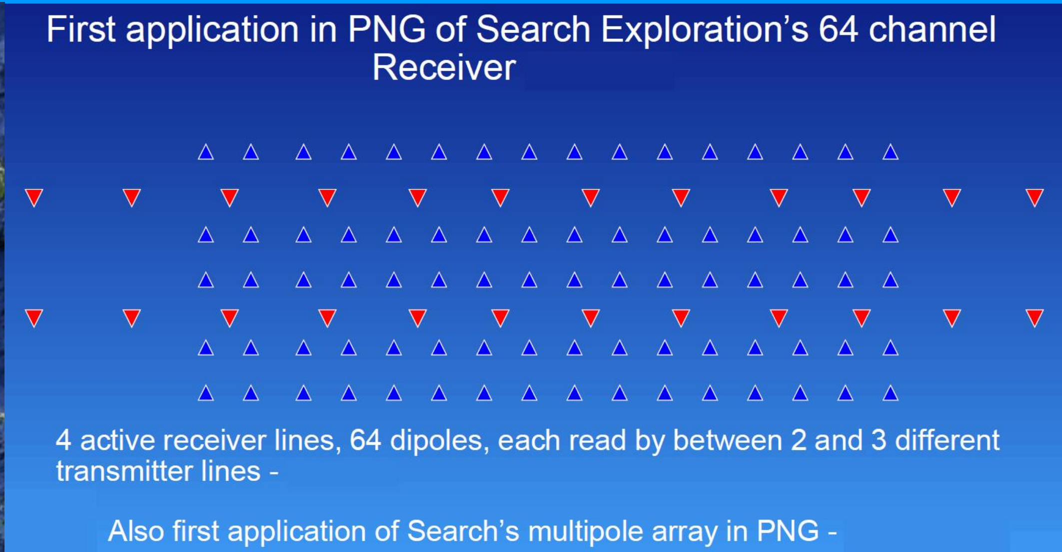
Burra copper deposit – vertical sections

Below are east-west vertical plots of the resistivity and I.P. resolution sections, and the resistivity VOI section approximately through the middle of the survey area. Note the region with significant I.P. resolution is shallower than the resistivity section. Also note the more complex pattern for the VOI plot with local artefacts.



Large field example - Tolukuma Au/Ag deposit, PNG

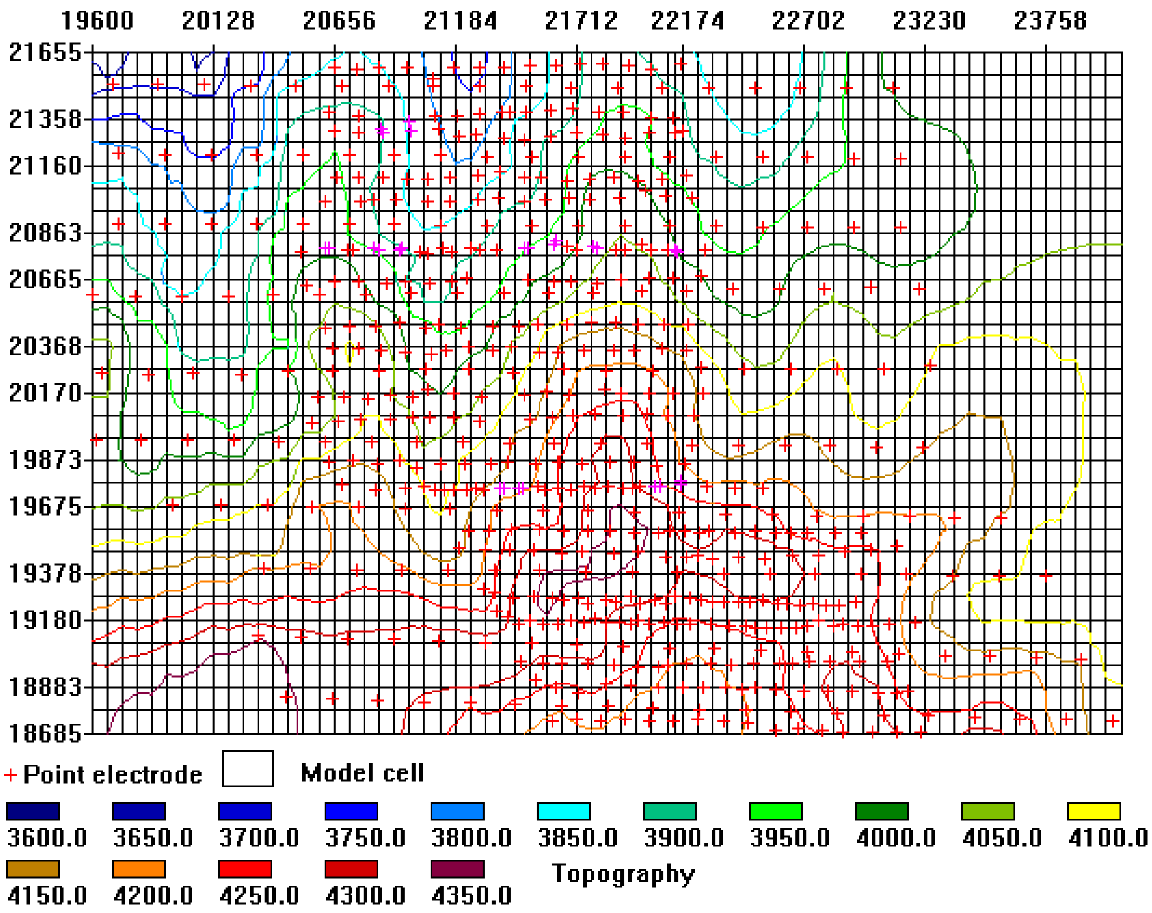
This is a low sulphidization epithermal Au/Ag deposit. Production is mainly from underground mines containing high grade narrow veins. The region of mineralization that is picked up by the geophysical surveys is wider. The survey area is very rugged. An offset survey configuration is used with transmitter and receivers on separate lines.



Tolukuma Au/Ag deposit – model grid

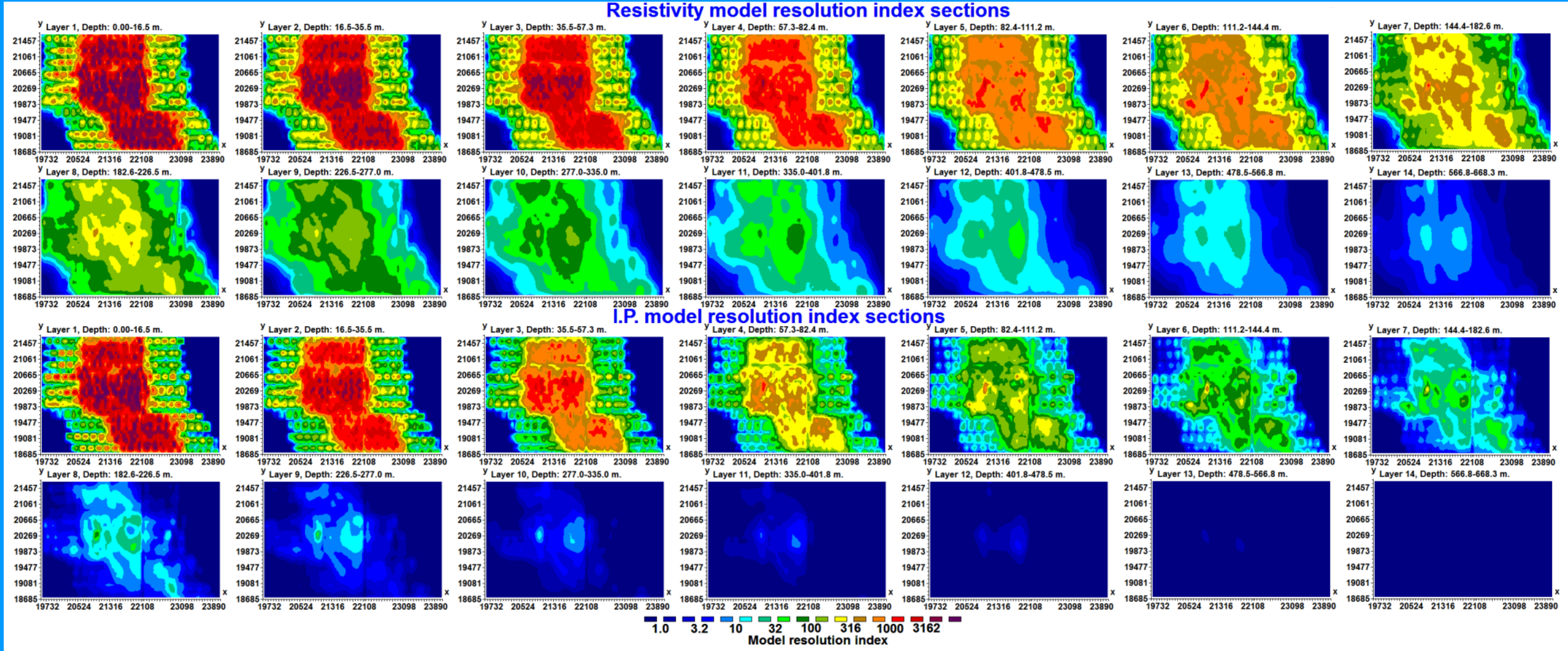
The model grid uses cells with widths of 66 and 99 m. in the x and y directions. The electrode spacings are closer near the centre, and wider at the sides.

There are 13519 data points, and the model has 28980 cells.



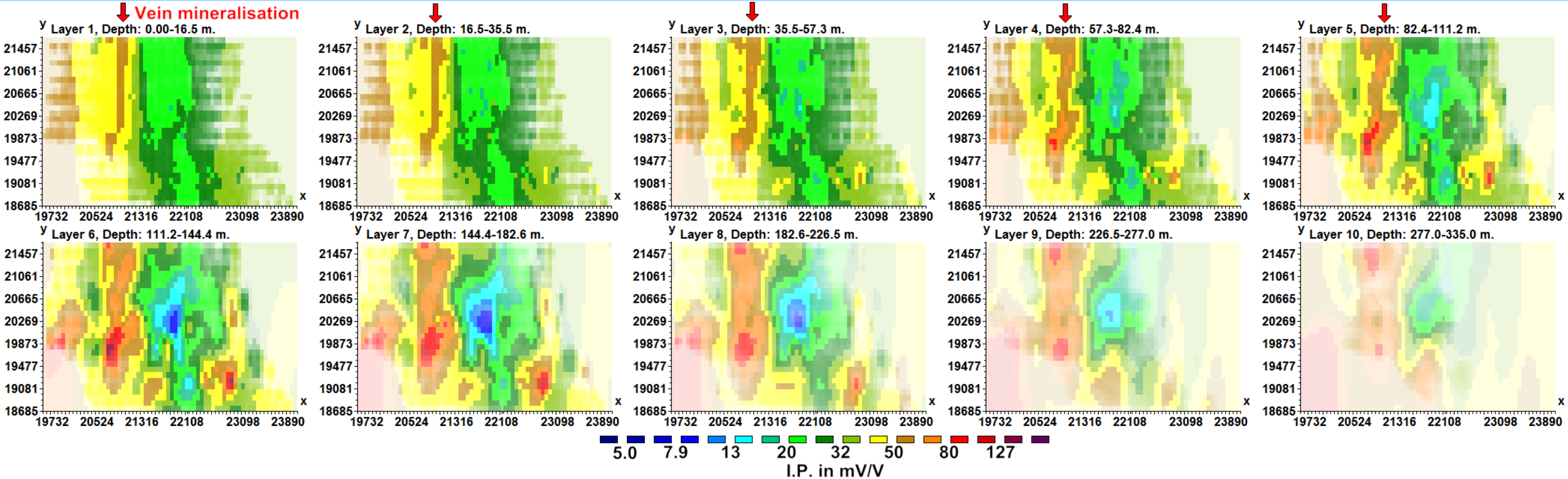
Tolukuma Au/Ag deposit – model resolution

There is a general decrease of resolution with depth with the highest values near the centre with more data coverage. The surveyed maximum depth is probably about 300-350 m. The I.P. resolution has a lower depth range.



Tolukuma Au/Ag deposit – I.P. model

The I.P. model is more diagnostic, with a N-S band of higher values near a known mineralised vein. The very high I.P values in the bottom layers are uncertain due to very low resolution values. In this plot, the models cells with low resolution values are blanked out.



Conclusions

1. Some method of assessing the reliability of the inversion results should be used.

2. The sensitivity method is the simplest but it is difficult to pick a consistent cutoff value.

3. The DOI/VOI method can be used for any data set where it is possible to carry out an inversion. The plot has sharp boundaries. A cut-off value of about 0.1 is usually used. However it is susceptible to local artefacts possibly caused by the inversion method used.

4. The model resolution method is more robust and less affected by the inversion settings used. However it is computationally more demanding which limits the size of the problem it can handle.