# Tweaking 3D inversion settings

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# Outline

1. Why inversion?
2. Inversion methods – global or local?
3. L1 or L2?
4. Banding effects
5. I.P. calculation methods
6. Intelligent design of 3-D arrays
7. Example and Conclusions

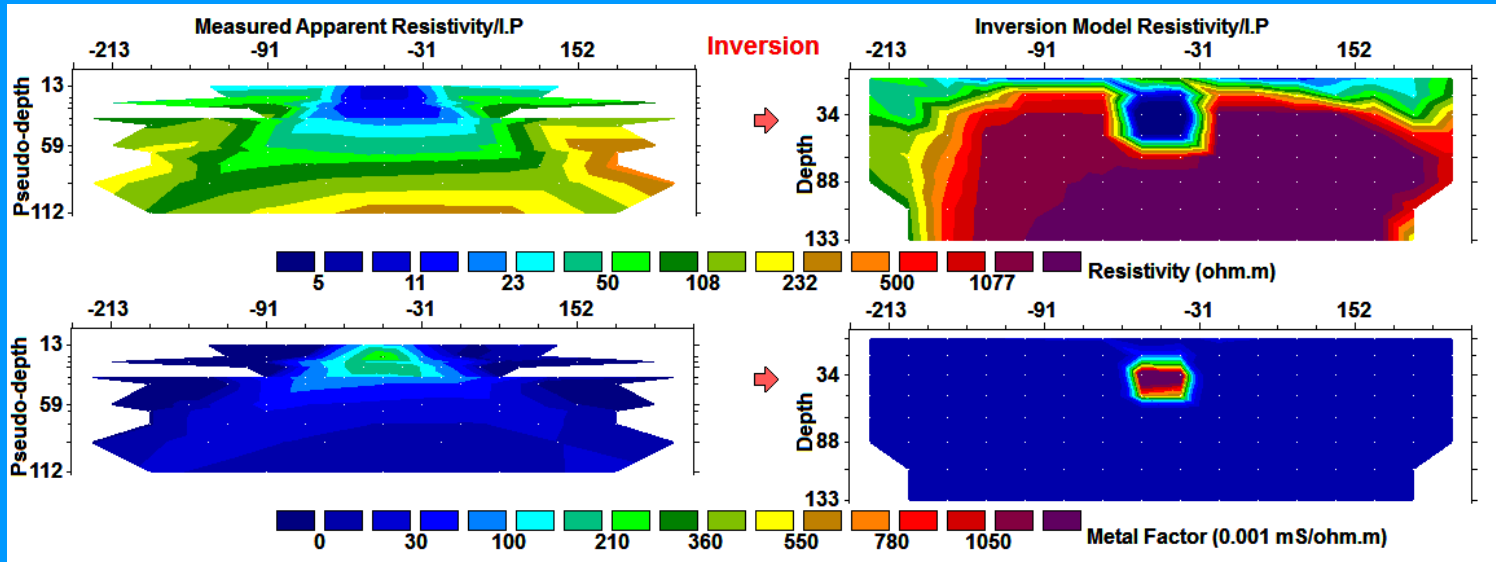
# Why inversion

**Apparent resistivity/I.P. measurements made in the field do not directly provide the information we want. They must be converted in a usable form :– inversion.**

**Data – What we measured in a survey. Apparent resistivity/I.P.**

**Model – Information about the subsurface. True resistivity/ I.P.**

**We want to find a model where the calculated response is ‘close’ to the measured data.**



# Inversion methods – global and local

**The relationship between the apparent resistivity/I.P. and the model resistivity/I.P. is complex and non-linear. An iterative method is used. Starting from an initial model, the model is automatically adjusted to minimize the data misfit (difference between calculated and measured apparent resistivity/I.P.) plus some measure of the model structure – an optimization problem. There are two general methods.**

1. **Global optimization methods (eg. simulated annealing). Advantage :- finds the 'best' possible solution. Disadvantage :- slow, requires many iterations (1000's) and forward models. Computational limits with PCs.**

2. **Local optimization methods (eg. conjugate gradients, Newton, least-squares). Advantage :- fast, usually converges in less than 10 iterations. Disadvantage :- non-optimal solutions, becomes trapped in a local minimum. Results depends on starting conditions.**

# Insufficient and inaccurate data

**The geoelectrical inversion problem is inherently non-unique, ambiguous and unstable. Data is limited and has noise. To stablise the inversion, assumptions must be made about the subsurface.**

1. **It is ‘smooth’, i.e. changes in a gradual manner, minimise model roughness.**
2. **Bias it towards models that are ‘closest’ to some reference model.**
3. **Other constraints – such as non-uniform weights across the model.**
4. **Noise distribution is L2 (minimise sum of squares, Gaussian) or L1(minimise absolute differences).**
5. **Find balance between model structure and data misfit (eg. L-curve method).**
6. **Data error known (eg. reciprocal measurements)? If yes, use it to ‘weigh’ the data. Usually not known for large scale 3-D I.P. surveys.**

# Least-squares inversion method

**The smoothness-constrained least-squares method is commonly used for large scale problems.**

$$\left[\mathbf{J}_i^T\mathbf{R}_d\mathbf{J}_i + \lambda_i\left(\mathbf{W}^T\mathbf{R}_m\mathbf{W} + \mu\mathbf{I}\right)\right]\Delta\mathbf{r}_i = \mathbf{J}_i^T\mathbf{R}_d\mathbf{g}_i - \lambda_i\left[\mathbf{W}^T\mathbf{R}_m\mathbf{W}\mathbf{r}_{i-1} + \mu\left(\mathbf{r}_{i-1} - \mathbf{r}_m\right)\right]$$

**F = roughness filters to stabilize inversion**

**$\lambda$ = roughness filter damping factor**

**$\mathbf{r}_{i-1}$ = previous iteration inversion model, $\Delta\mathbf{r}_i$ = change in model resistivity/I.P.**

**$\mathbf{r}_m$ = reference model, $\mu$ = reference model damping factor**

**g = difference between calculated and measured data (data misfit)**

**J = Jacobian matrix of partial derivatives.**

**W= roughness filter, $\mathbf{R}_d$,$\mathbf{R}_m$= weights used for L1-norm**

**The finite-difference or finite-element method is used to calculate the model response and Jacobian matrix.**

**The linear conjugate gradient is used to solve the least-squares equation.**

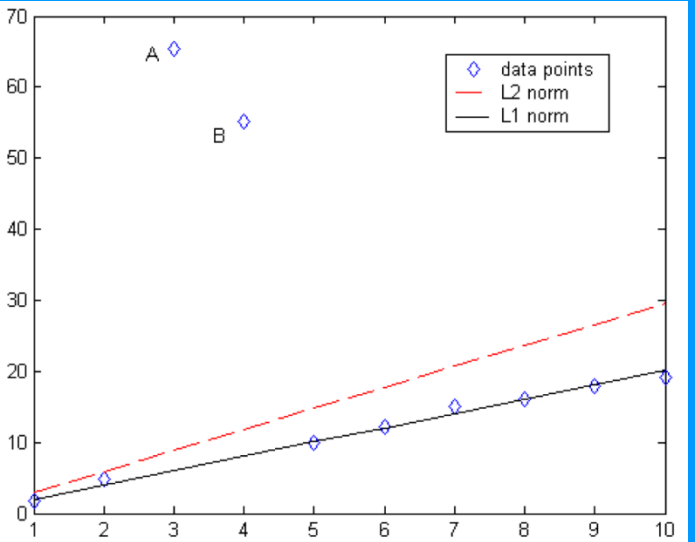
# L1 or L2 for data misift?

**The $L_2$ norm minimises the sum of squares of the data misfit, the L1 norm minimises the sum of the absolute values.**

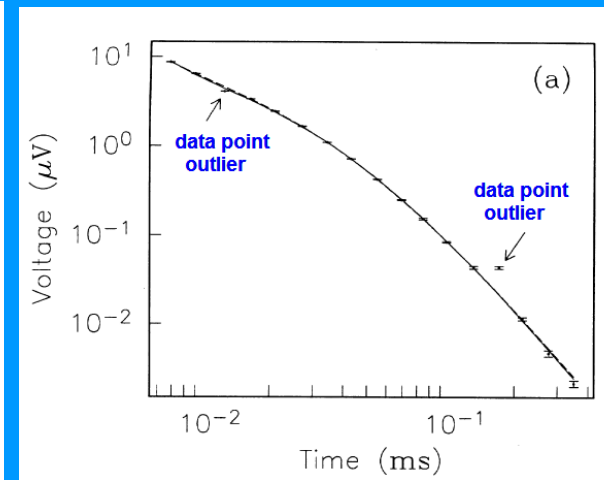
$$S_2 = \sum (\rho_c - \rho_m)^2 \,,\; S_1 = \sum |\rho_c - \rho_m|$$

**The L2 norm is optimal if the data misfit has a 'Gaussian' distribution, but is off when there are data outliers. The L1 norm is less sensitive to outliers.**

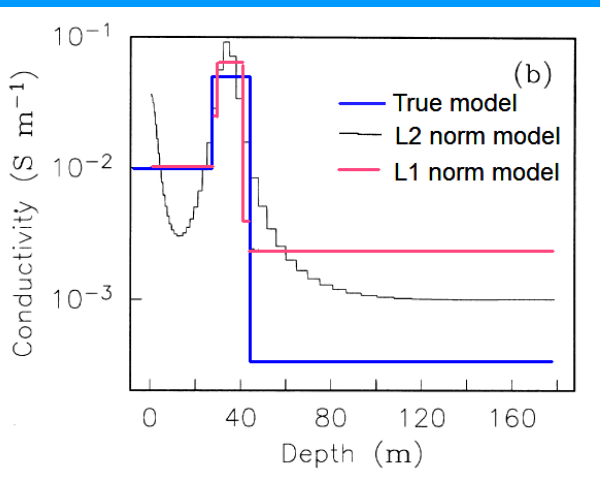
## Line fitting example




## 1-D inversion example



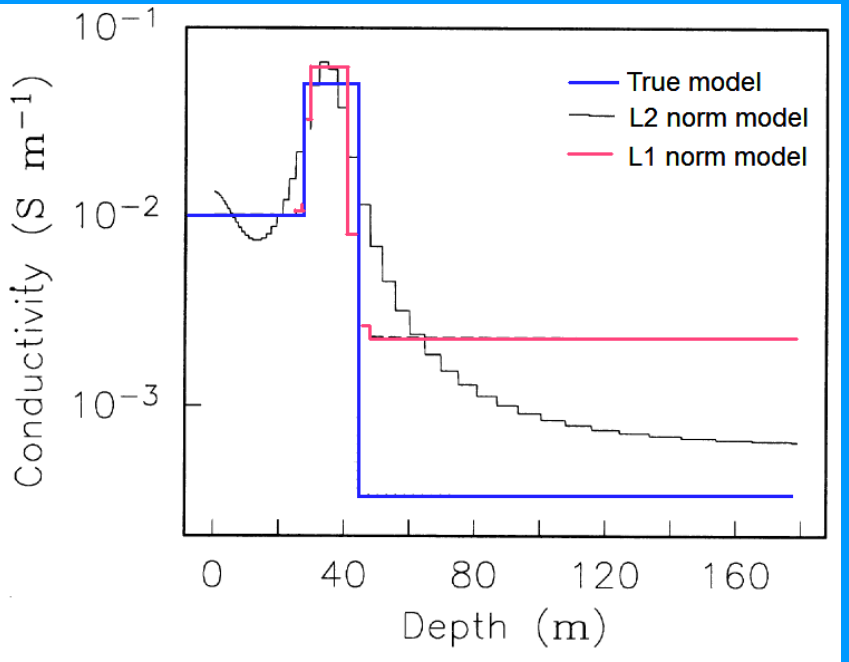

Data with noise added at two points

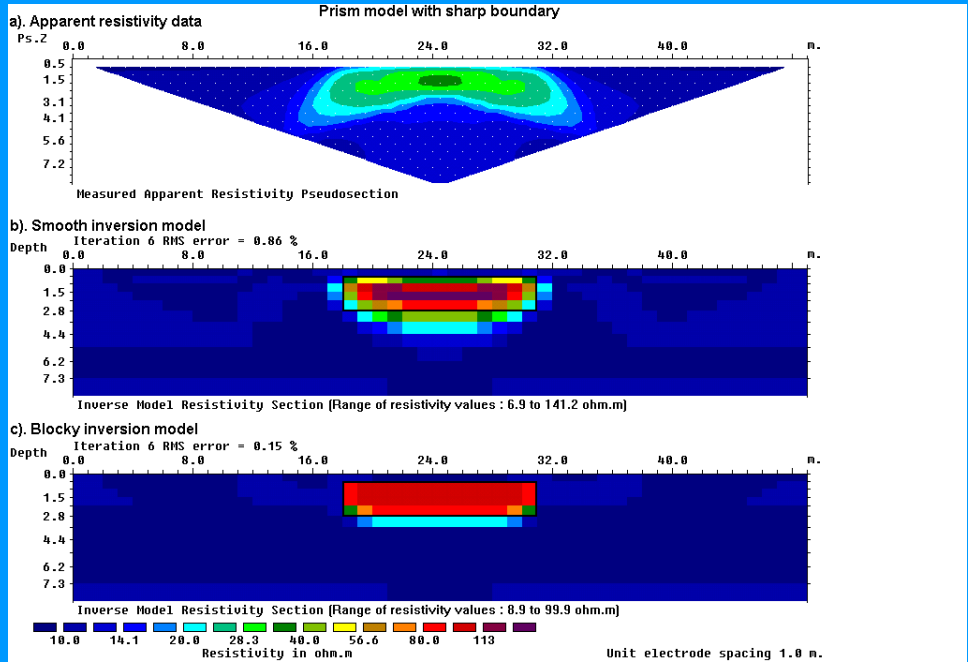



Models with L1 and L2 norms used for model roughness and data misfit.

# L1 or L2 for model roughness?

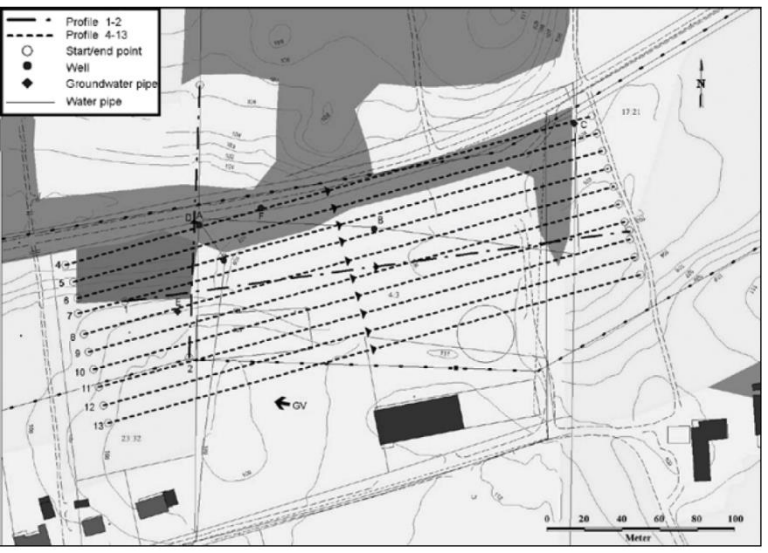
**The $L_2$ norm method minimizes the sum-of-squares of the change in the model resistivity values ($W^TW$ term). It gives accurate results where the subsurface resistivity changes in a gradual manner, but results are less accurate when there are sharp boundaries. The $L_1$ norm method attempts to minimise the absolute value of the model changes, and tends to give a model with sharper transitions.**

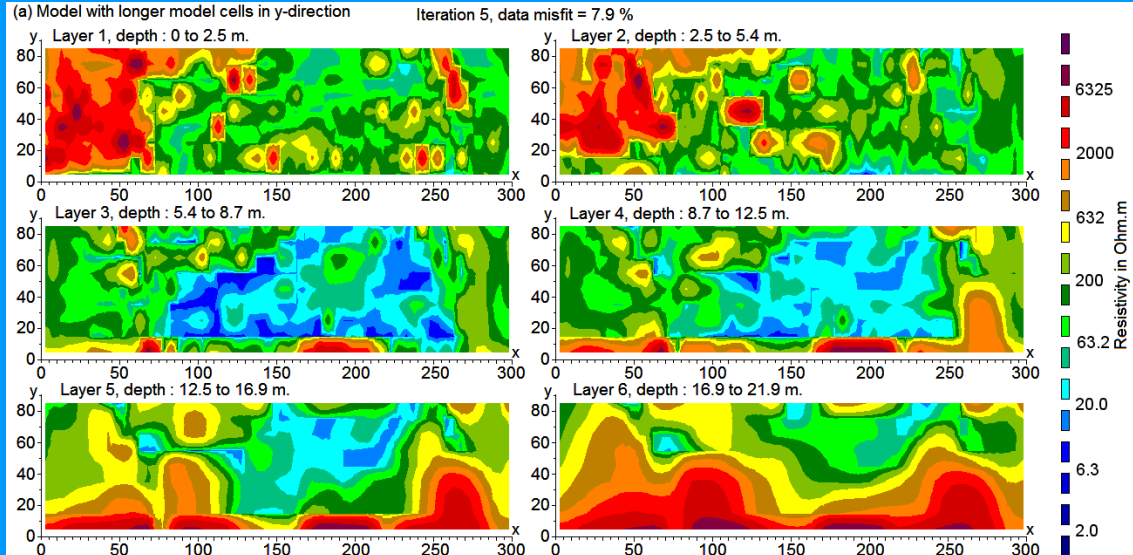


# 3-D data sets and banding effects

**Many '3-D' data sets are collated from 2-D survey lines. The inversion models frequently show artefacts aligned along the direction of the lines. The example (Ekeboda landfill survey, Sweden) below has 10 parallel 2-D lines. The 3-D model shows prominent linear artefacts that are aligned along (or perpendicular to) the direction of the survey lines. This effect is particularly prominent when the L1-norm is used for the model roughness filter.**

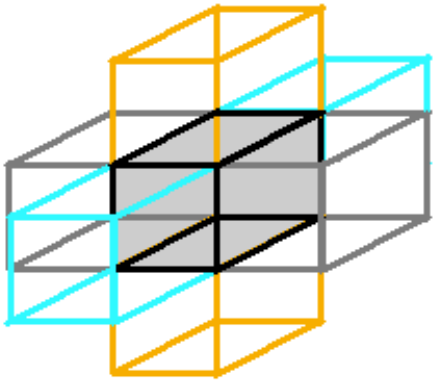


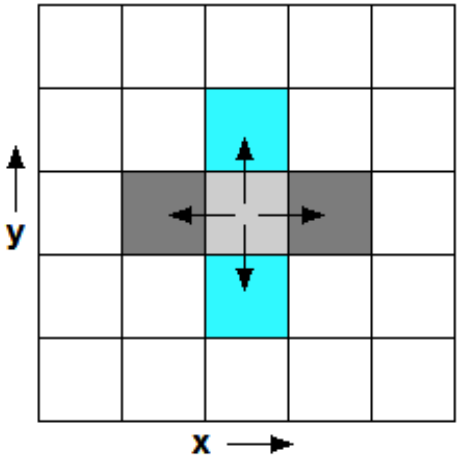
# Diagonal roughness filter

The artefacts are due to the roughness filter that tries to minimise changes in the x, y and z directions. To reduce the banding effects, the roughness filter is modified with components in the diagonal directions as well.

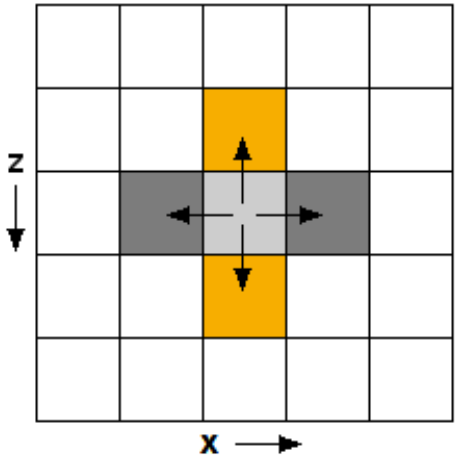
Coupling of 3-D model cells in roughness filter



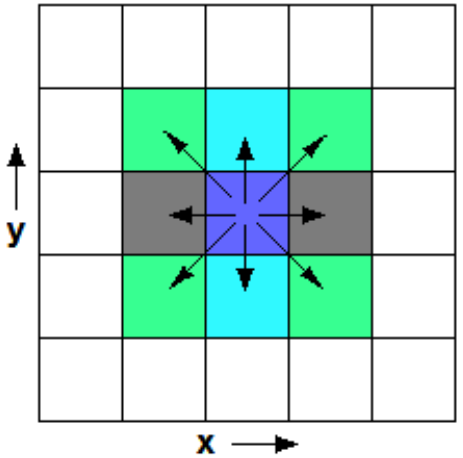
Normal horizontal roughness filter x and y components




Normal vertical roughness filter with z component

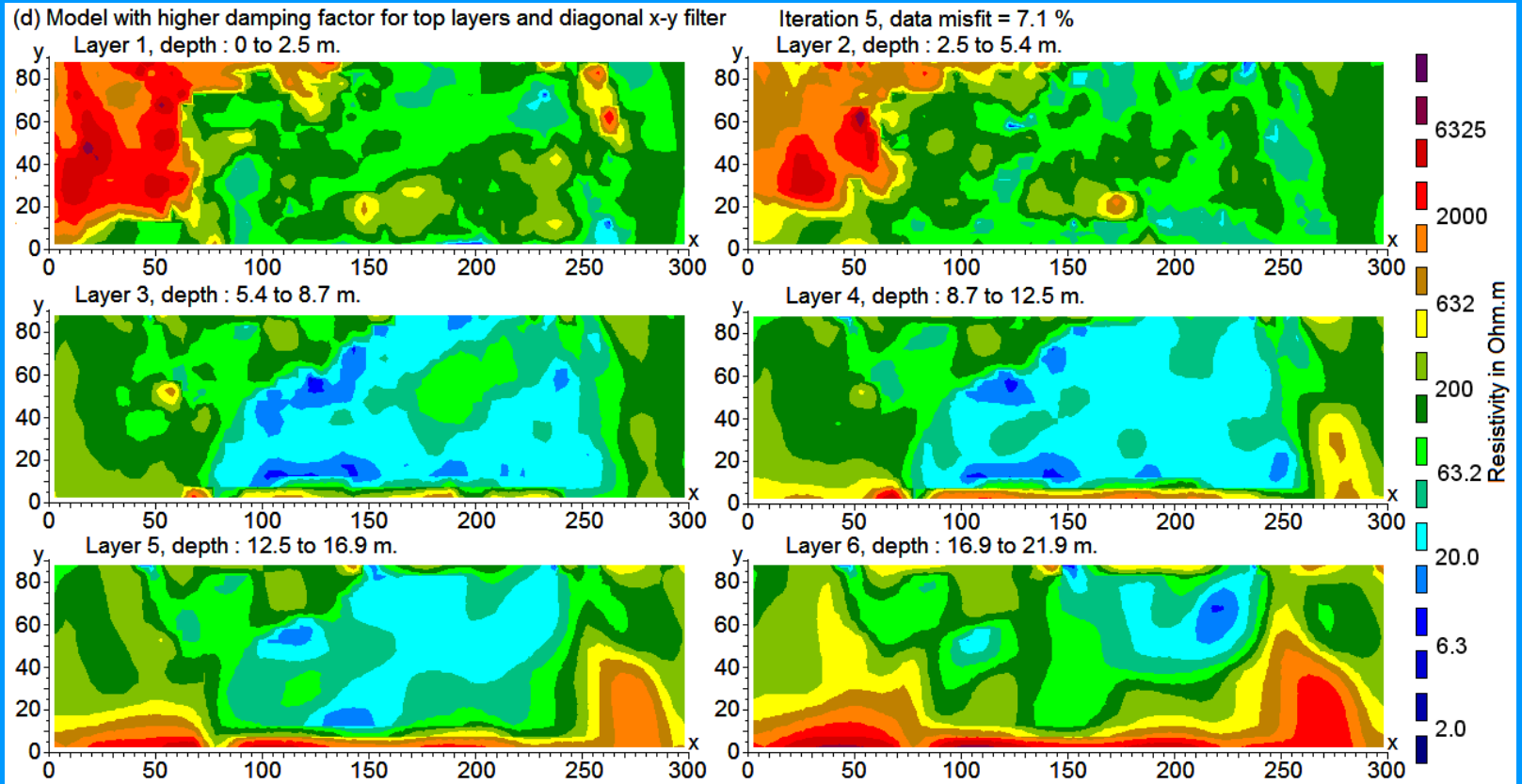



Horizontal roughness with diagonal x-y components



# Ekeboda landfill data set – model with diagonal filters

The elongated structures in the *x* direction are almost completely removed when a roughness filter with diagonal components is used.



# Apparent I.P. calculations methods

The two methods are the **perturbation** and **complex resistivity** approaches. The **perturbation** method considers the I.P. (**m**) as a 'small' change from a base resistivity model ($\sigma_{DC}$) giving an effective conductivity $\sigma_{IP}=(1 - m)\sigma_{DC}$ . The apparent I.P. is calculated from the potentials ($\phi$) of two forward models.

$$m_a = [\phi(\sigma_{IP}) - \phi(\sigma_{DC})] / \phi(\sigma_{DC})$$

The conductivity is treated as a complex quantity in the **complex resistivity** method, $\sigma = \sigma_{DC} - i\ m.\sigma_{DC}$. $\sigma_{DC}$ is the DC conductivity, while **m** is in the imaginary part. A complex potential is then calculated.

$$\phi = \phi_r + i\ \phi_i$$

The apparent resistivity value is calculated by using the amplitude of the complex potential, $\phi_A = (\phi_r^2 + \phi_i^2)^{0.5}$. The apparent chargeability is calculated using the ratio of the imaginary and real components, $m_a = \phi_i / \phi_r$ .

# Problems with perturbation method

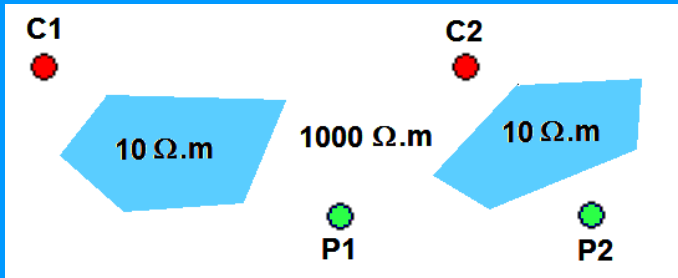
**(1). Its is based on the assumption the chargeability is 'small'.**

**(2). The method used to calculated the apparent I.P.:-**

$$m_a = [\ \phi(\sigma_{IP}) - \phi(\sigma_{DC})\ ] / \phi(\sigma_{DC})$$

**is sensitive to errors in the forward modeling method, particularly when $\phi(\sigma_{DC})$ is small. In some situations it might even be negative, particularly with the new 'offset' type of arrays. In such cases, the calculated $m_a$ values are not reliable.**

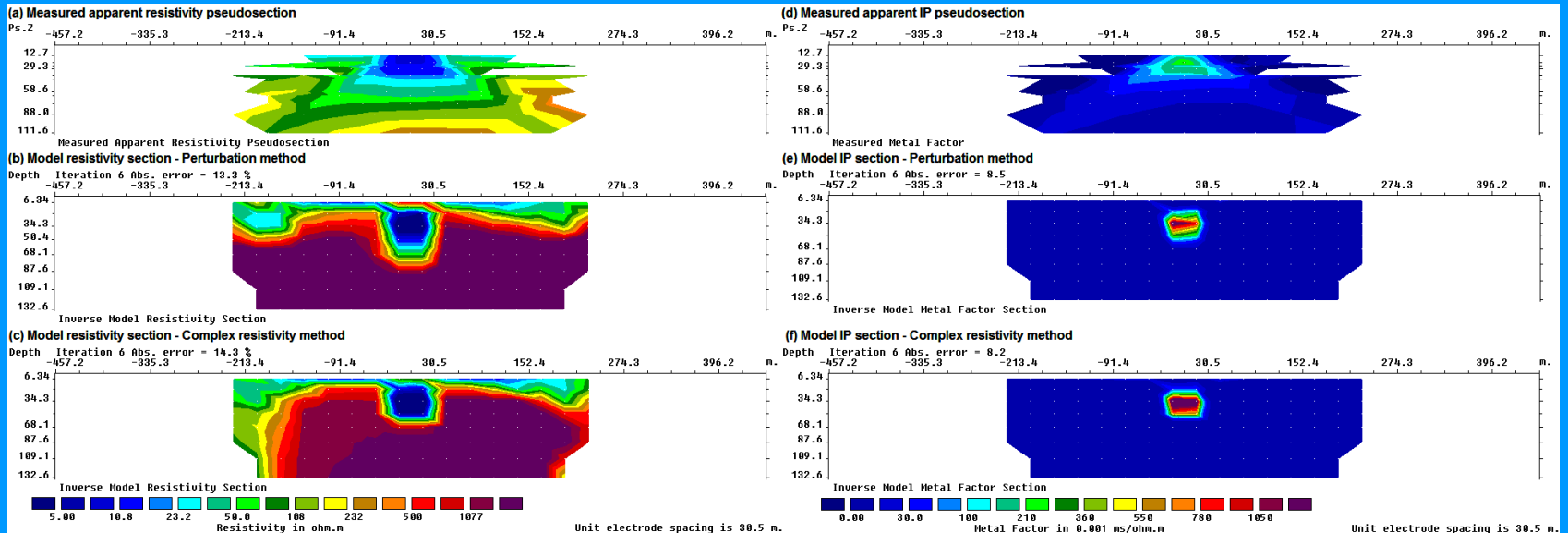
**The accuracy of $m_a$ does not depend on the D.C. potential accuracy in the complex resistivity method. However, it requires twice as much memory as the perturbation method which can be a limitation for large 3-D models.**



# Comparisons between complex resistivity and perturbation methods

**Below is an example for a massive sulphide orebody. For low I.P. contrasts and 'well behaved' arrays both methods give similar results, with the complex resistivity method sometimes giving sharper images.**

**For 'extreme' offset pole-dipole and dipole-dipole type of arrays, the complex resistivity method is sometimes the only viable option. More memory is required, but RAM is relatively inexpensive.**



# Intelligent design of 3-D arrays

**With the popularity of the offset pole-dipole array as an efficient 3-D survey method, and new multi-channel systems, there has been a profusion of the offset type of arrays. Some of these arrays were designed to give wider data coverage, but proven to be impractical for data interpretation. The following criteria are proposed to filter out potentially 'unstable' arrays.**

1. **Extreme geometric factors. If it is more than a few million, the potential is likely to be too small (microvolts?). I.P. potential ~ 1% DC potential.**

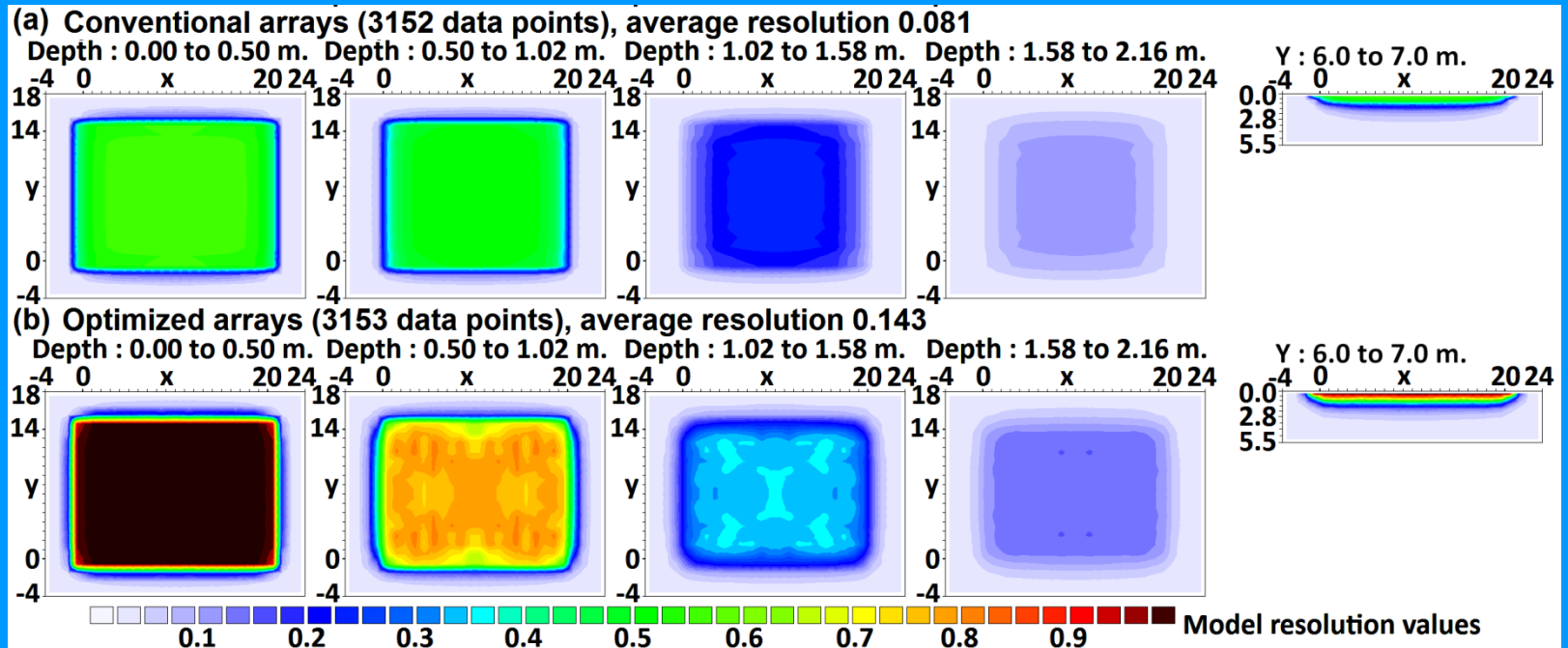
2. **Geometric factor (*K*) sensitivity to errors in the electrodes positions (*s*) for offset type of arrays with electrodes along different lines. Due to the finite accuracy of the forward modelling routine, they might end up with negative apparent resistivities.** $$s^2 = \left(\frac{\partial K}{\partial A}\right)^2 + \left(\frac{\partial K}{\partial B}\right)^2 + \left(\frac{\partial K}{\partial M}\right)^2 + \left(\frac{\partial K}{\partial N}\right)^2$$

3. **Information content. Does the array give significant extra information not already given by existing arrays in the data set?**

# Optimised 3-D arrays using the model resolution

**To quantify the information content we calculate the model resolution for a homogenous half-space for the set of arrays. Below are plots of the model resolution for a 21x15 survey grid for two data sets, one with conventional dipole-dipole and Wenner-Schlumberger arrays and the second with non-conventional arrays optimised to give the maximum resolution.**

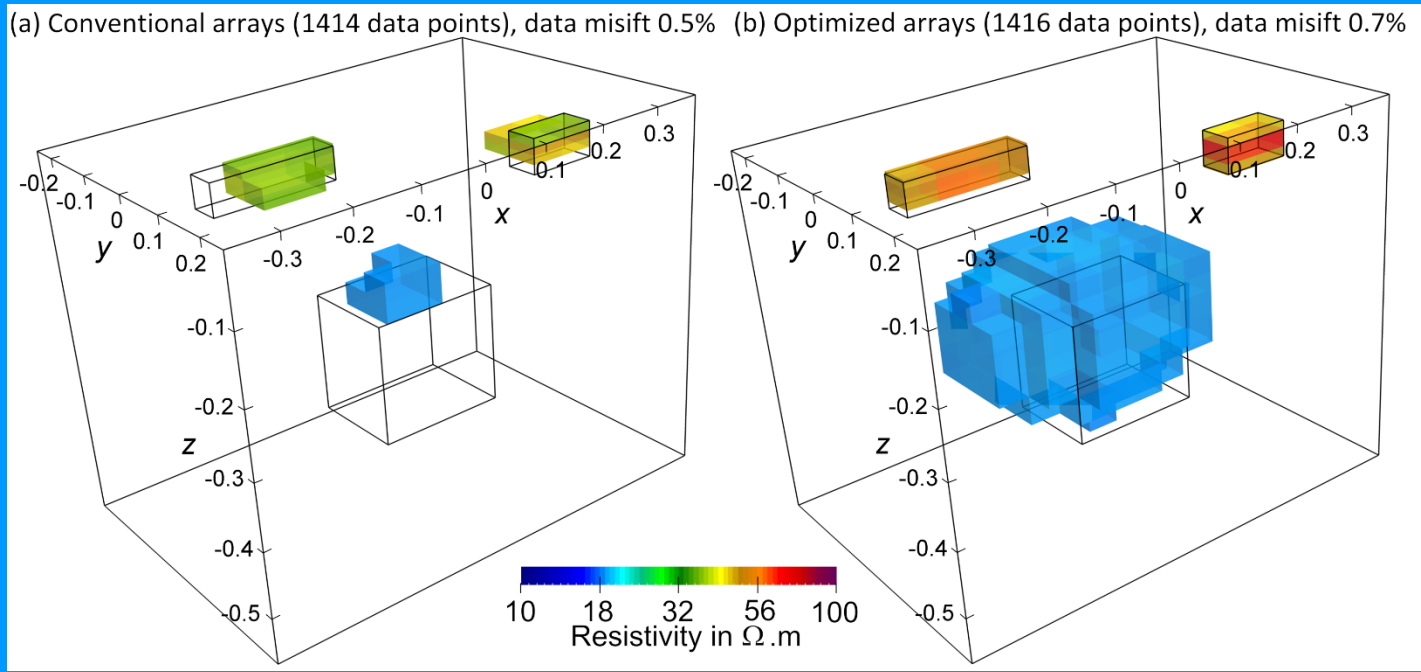
$$\mathbf{R} = \left[\mathbf{J}^{\mathrm{T}}\mathbf{J} + \lambda\mathbf{F}\right]^{-1}\mathbf{J}^{\mathrm{T}}\mathbf{J}$$



# Example of optimised 3-D arrays

**The results from an experimental data set measured in a tank using a 16 by 6 electrodes survey grid layout is shown. The tank was filled with water (15 Ωm) and 3 rectangular plastic blocks were placed at different depths. A conventional array (dipole-dipole and Wenner-Schlumberger) was compared with arrays optimised using the model resolution. Values of above 36 Ωm in the vicinity of blocks A and B are shown.**

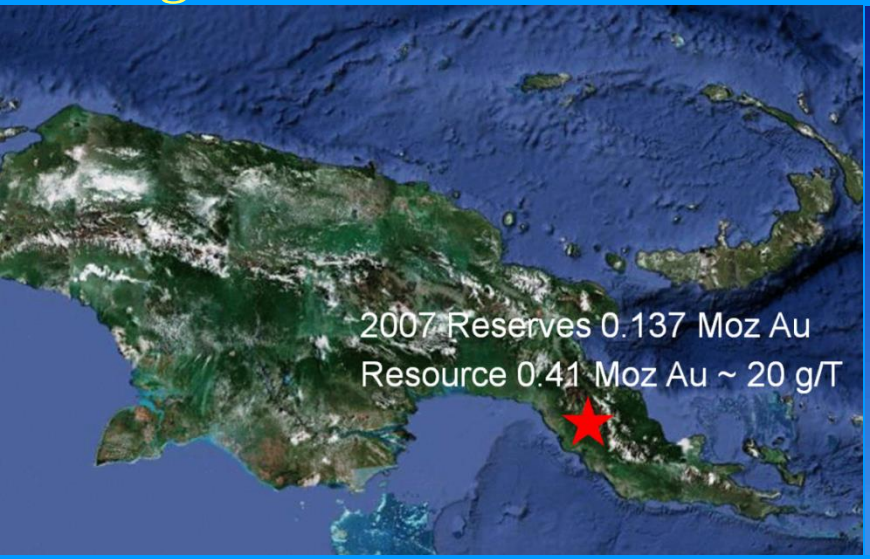
**(above 18 Ωm for block C).**

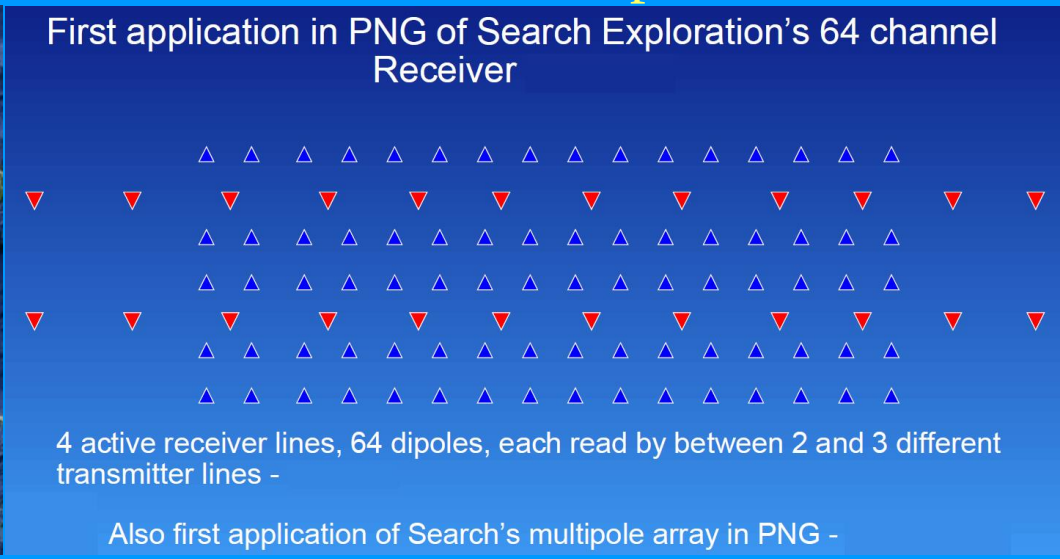



Loke, M.H., P. B. Wilkinson, P.B., Uhlemann, S.S., Chambers, J.E. and Oxby, L. S., 2014. Computation of optimized arrays for 3-D electrical imaging surveys. Geophysical Journal International, **199**, 1751-1764.

# Model reliability example - Tolukuma Au/Ag deposit

Some means to determine regions of the inversion models that are well resolved is required to separate fact from fiction. The example is from a 3-D survey to map a Au/Ag deposit found in the form of high grade narrow veins in PNG. The survey area is very rugged. An offset survey configuration is used with transmitter and receivers on separate lines.



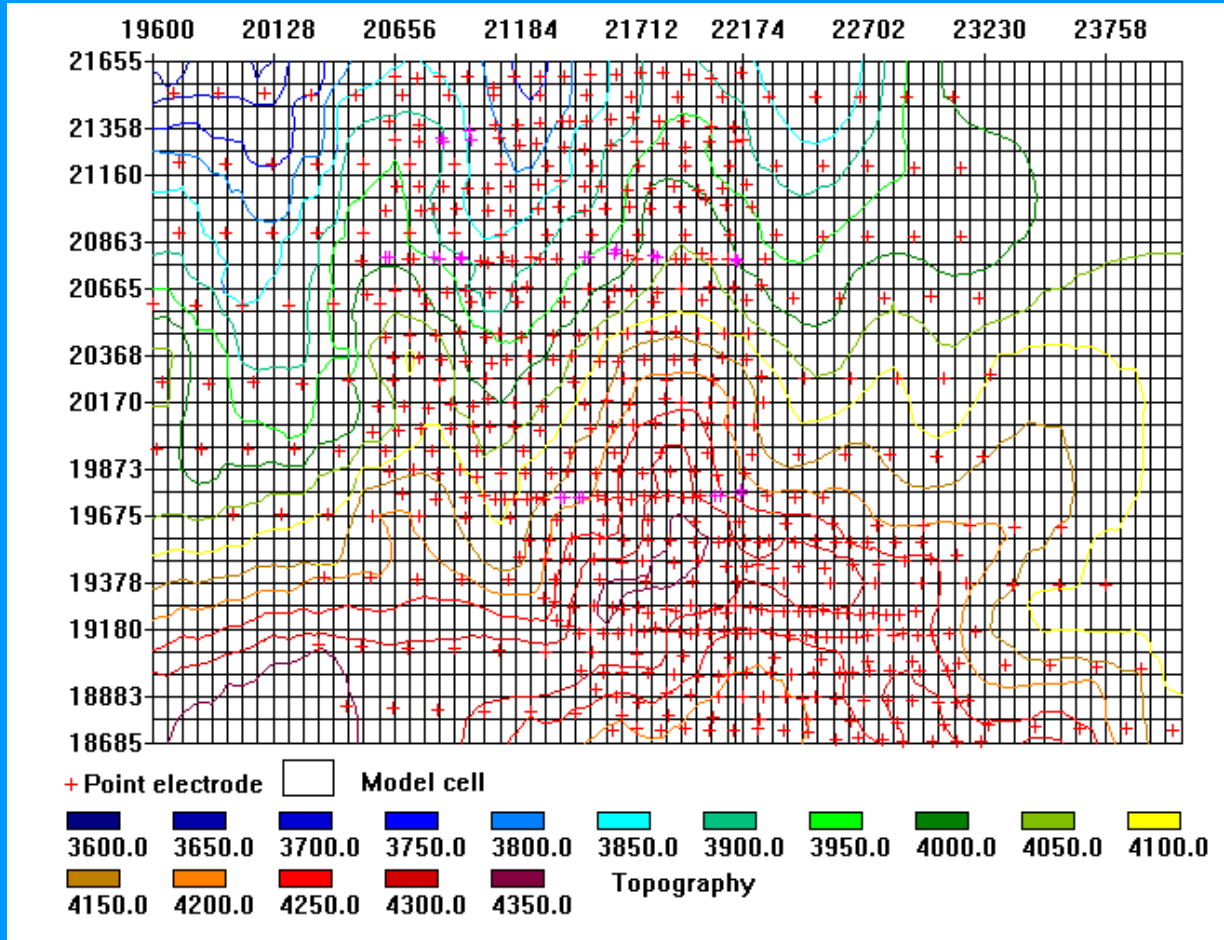




M.H. Loke, K. Frankcombe and D. F. Rucker, 2013. The inversion of data from complex 3-D resistivity and I.P. surveys. 23rd International Geophysical Conference and Exhibition Program (ASEG-PESA 2013).

# Tolukuma Au/Ag deposit – model grid

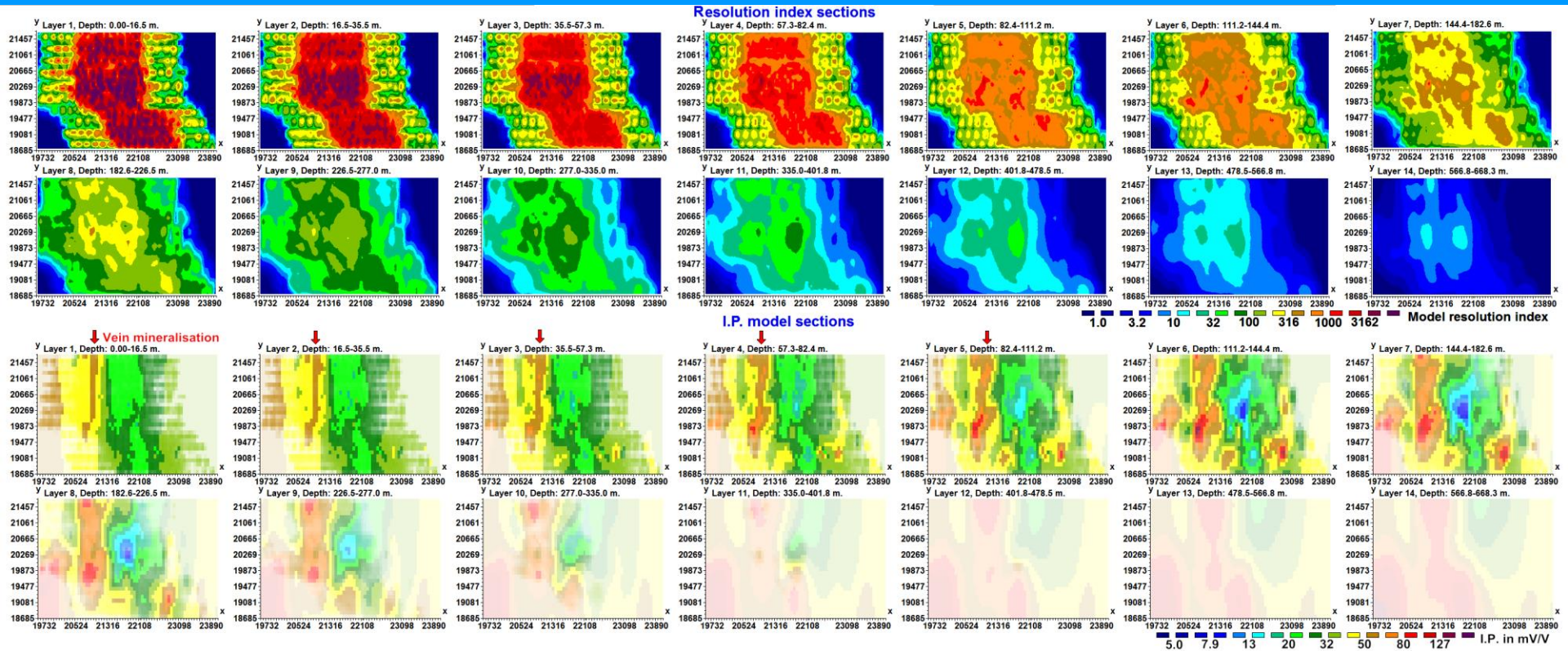
The model grid uses cells with widths of 66 and 99 m. in the x and y directions. The electrode spacings are closer near the centre with better data coverage, and wider at the sides.

There are 13519 data points, and the model has 28980 cells.



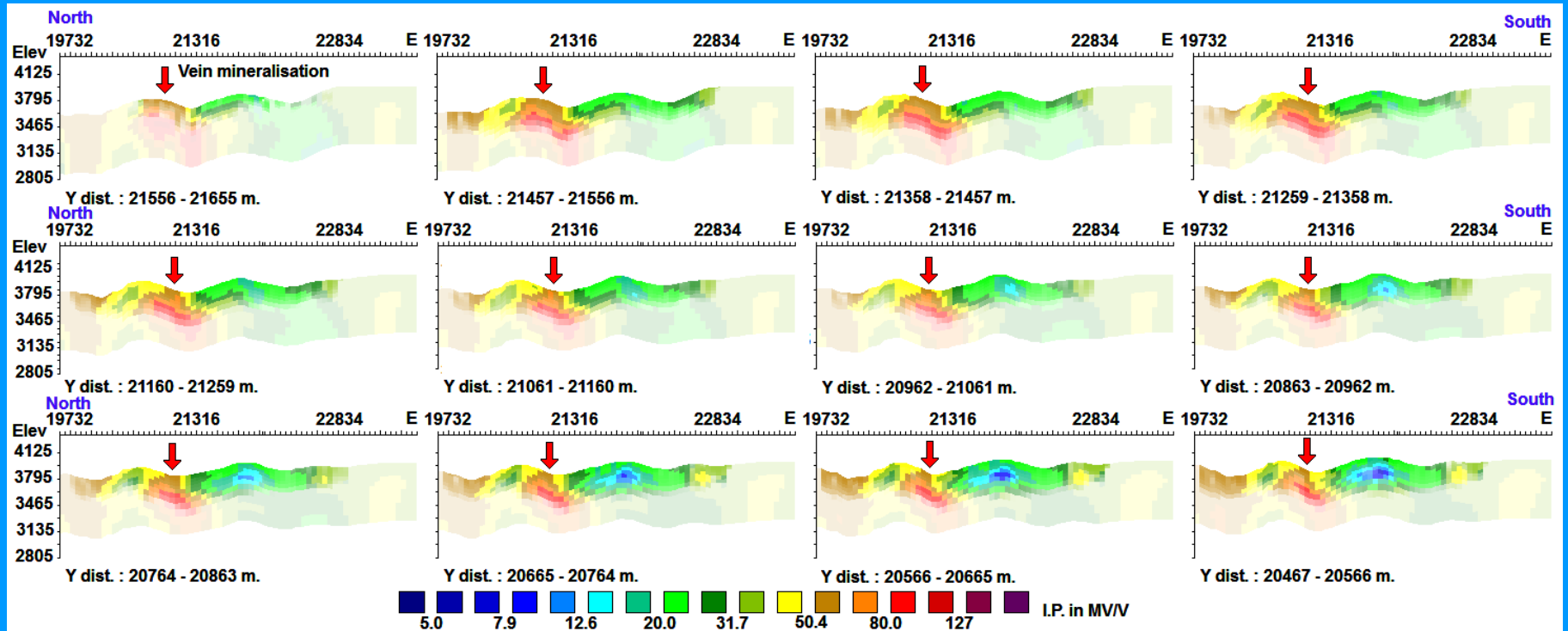
# Tolukuma Au/Ag deposit – model resolution

**A plot of the resolution and I.P. (with gradual fade out for low resolution areas) horizontal model sections are shown. There is a band of high I.P. values in an area with known mineralization located within a region of significant resolution.**



# Tolukuma Au/Ag deposit – I.P. model with topography

**This shows EW vertical sections of the I.P. model, with the northernmost section at the top-left. It shows more clearly the relationship between the band with higher I.P values with the surface location of the vein. Sulphides around a gold vein were encountered in a 500 m. deep borehole by Petromin.**



# Conclusions

**3-D resistivity/I.P. inversion is inherently non-unique with limited and noisy data. A number of constraints are used to stablise the inversion. The constraints used should reflect the true geology or data quality.**

**If data outliers are present, the L1-norm data constraint can reduce its effect.**

**Choice of the L2 or L1-norm model roughness filter should depend on whether the expected geology/resistivity varies in a 'smooth' or 'blocky' manner.**

**For '3-D' data collated from 2-D survey lines, diagonal roughness filters should be used to reduce directional bias in the model.**

**For data sets with 'extreme' unstable arrays, the complex resistivity method can be used at the expense of larger computer memory requirements.**

**Arrays used should be designed by filtering out those with large geometric factors and sensitive to electrodes positions errors, and those that do not add much information content.**

**Some estimate of the model reliability, such as VOI or model resolution, should be made.**