Time-lapse 4-D resistivity imaging inversion with positivity constraints

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Outline

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2. Example of conventional time-lapse inversion
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4. Landfill survey (Sweden) – modified positivity constraints
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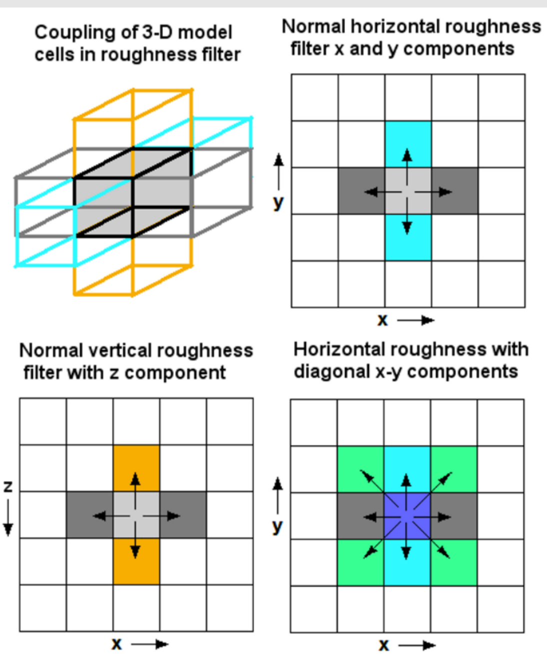
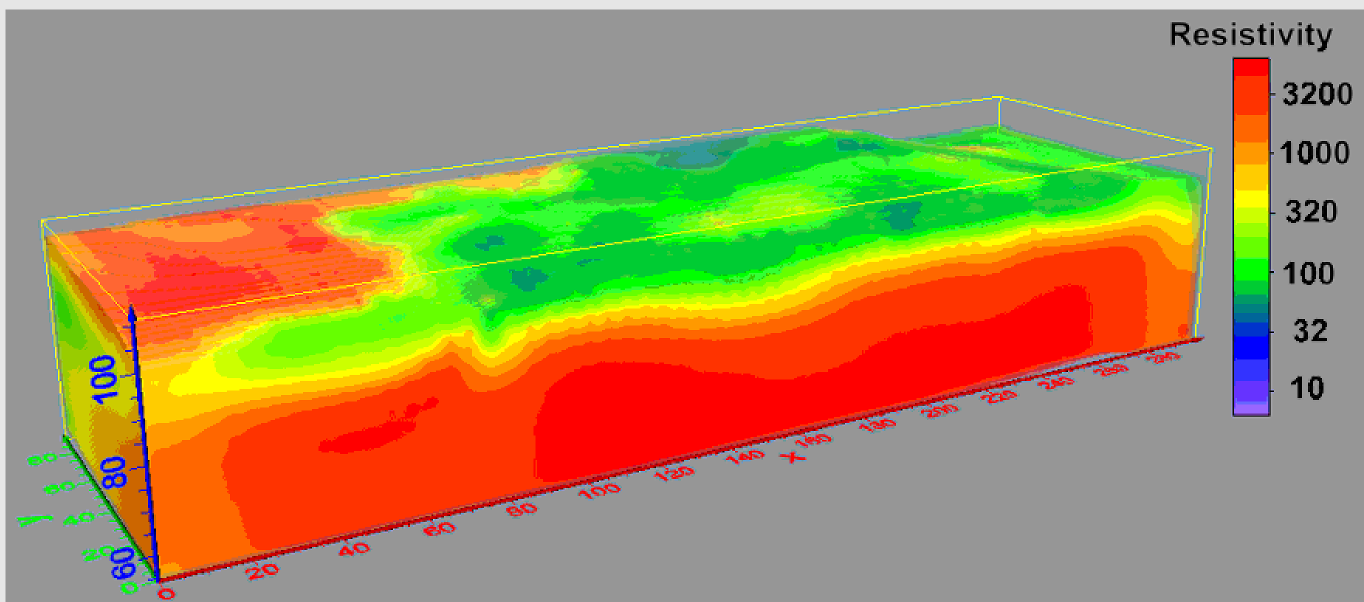
2-D and 3-D ERT surveys and data inversion

Over the last 25 years, 2-D and 3-D ERT surveys have been widely used. The equation for the 2-D and 3-D smoothness-constrained least-squares inversion method is given by

$$\left[\mathbf{J}_i^T \mathbf{R}_d \mathbf{J}_i + \lambda_i \mathbf{W}^T \mathbf{R}_m \mathbf{W}\right] \Delta \mathbf{r}_i = \mathbf{J}_i^T \mathbf{R}_d \mathbf{g}_i - \lambda_i \mathbf{W}^T \mathbf{R}_m \mathbf{W} \mathbf{r}_{i-1}$$

The roughness filter **W** minimizes the change the resistivity between adjacent model cells in the x, y and z directions. λ is a damping factor, Δ**r** is the change in model resistivity values, and **g** is the data misfit of the current model. **J** is the Jacobian matrix of partial derivatives. $\mathbf{R}_d$ and $\mathbf{R}_m$ are weighting matrices for the L1-norm inversion method.



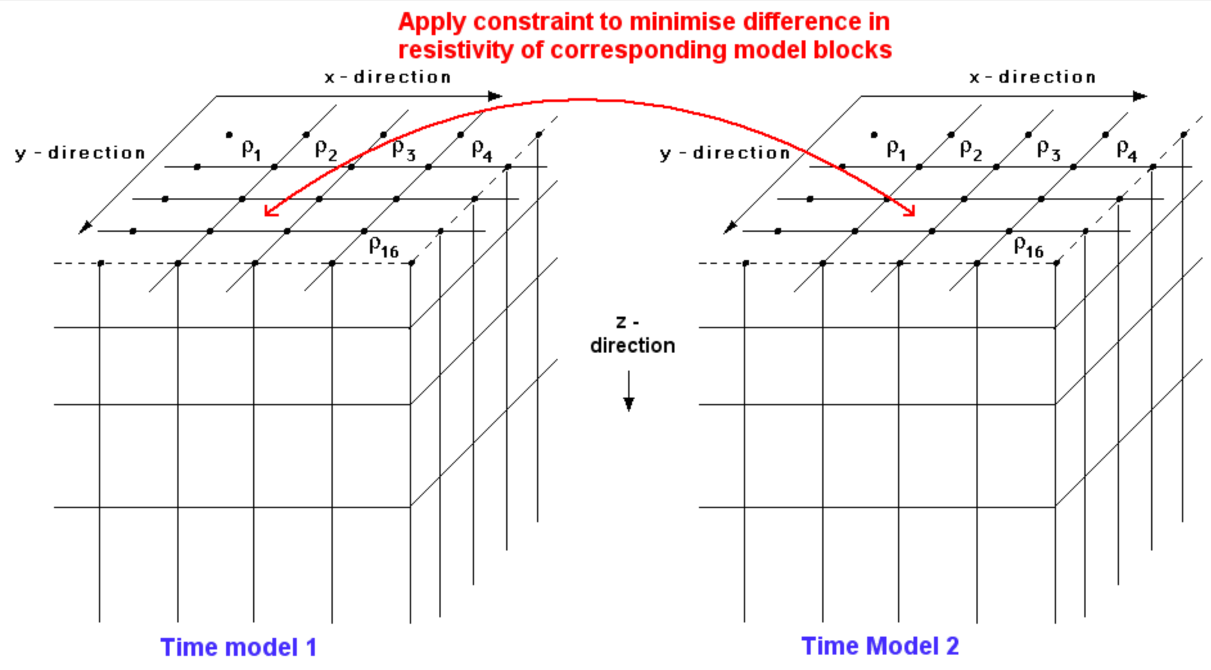


Smoothness constrained 4-D time-lapse inversion method

In 4-D inversion, the smoothness-constrained least-squares equation is modified to include a time-lapse constraint.

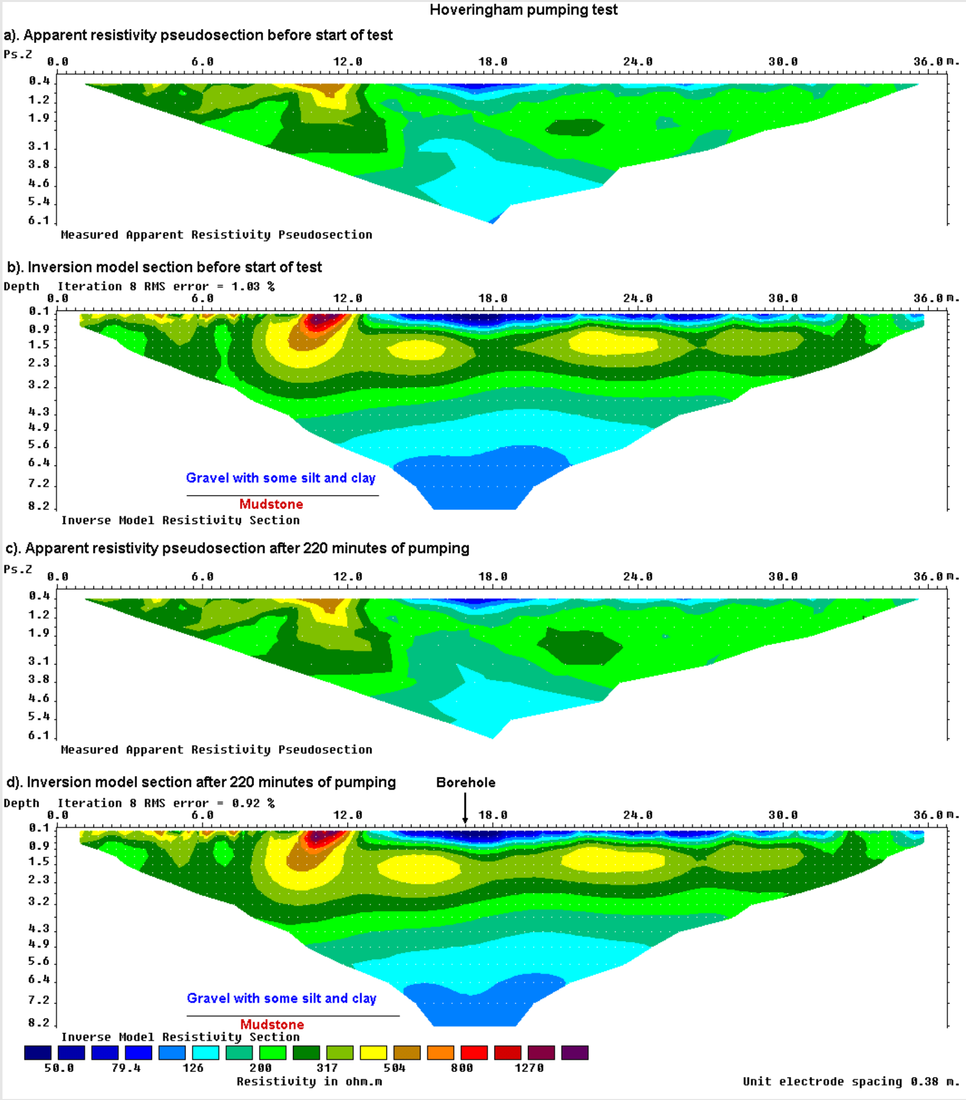
$$\left[\mathbf{J}_i^T\mathbf{R}_d\mathbf{J}_i + \lambda_i\left(\mathbf{W}^T\mathbf{R}_m\mathbf{W} + \alpha\mathbf{M}^T\mathbf{R}_t\mathbf{M}\right)\right]\Delta\mathbf{r}_i = \mathbf{J}_i^T\mathbf{R}_d\mathbf{g}_i - \lambda_i\left(\mathbf{W}^T\mathbf{R}_m\mathbf{W} + \alpha_i\mathbf{M}^T\mathbf{R}_t\mathbf{M}\right)\mathbf{r}_{i-1}$$

M is the difference matrix applied across the time models. $\mathbf{R}_t$ is used to select between the L1 and L2 norm time-lapse constraint. α is the temporal damping factor that gives the relative weight for minimizing the temporal changes. It reduces artifacts due to noise using the cross-model damping factor, but it does not directly constrain the direction of the changes of the resistivity with time (i.e. whether the resistivity increases or decreases).



Conventional time-lapse example : Pumping test, U.K.

This survey was from an experiment to map the change in the aquifer during a pumping test in Hoveringham, England. Water was pumped for about 220 minutes from a borehole. Measurements were made before, during and after the pumping.



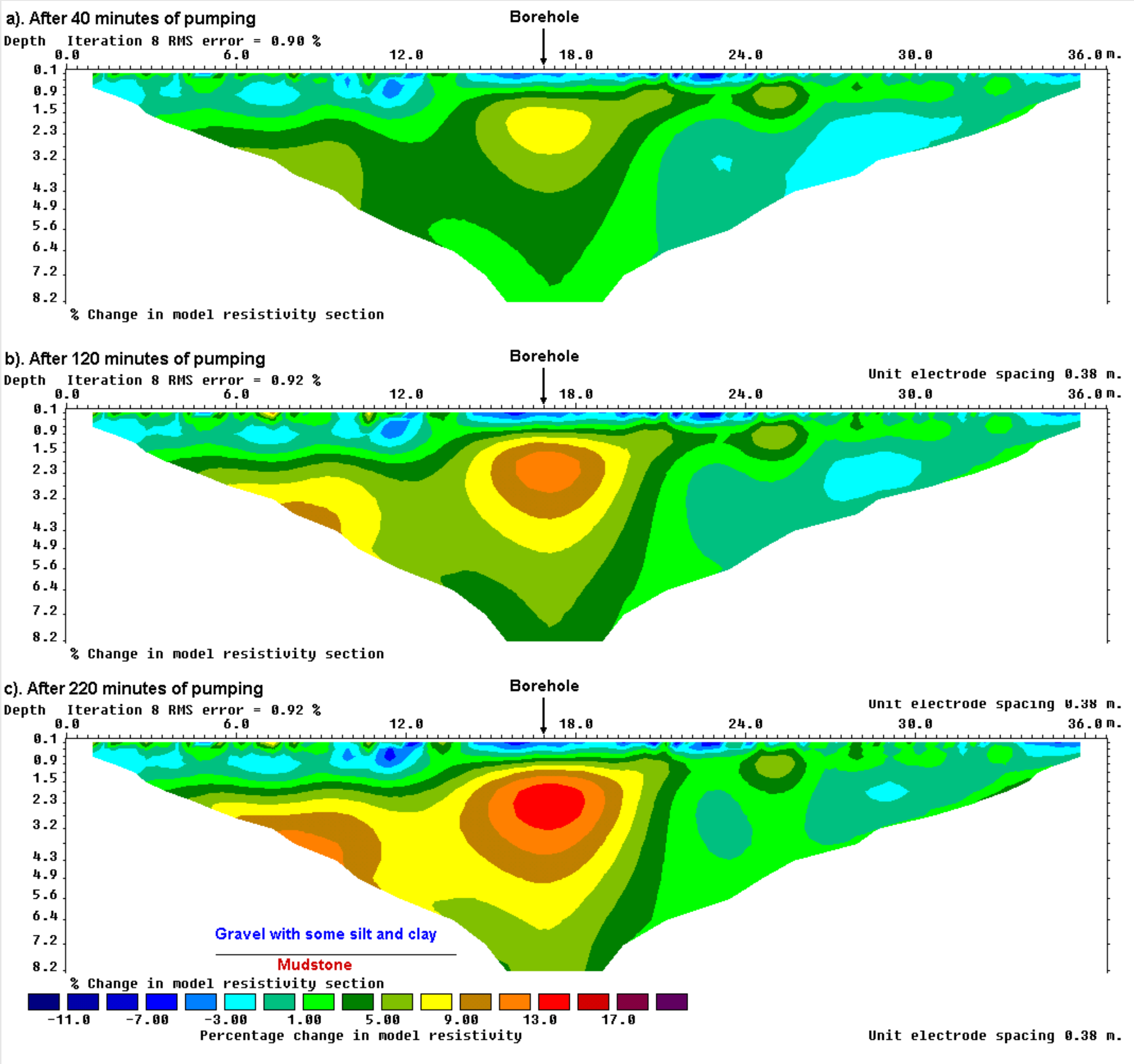
- **(a) Apparent resistivity pseudosection and**
- **(b) inverse model for data set before the start of the pumping test.**
- **(c) Apparent resistivity pseudosection and**
- **(d) inverse model for data set after 220 minutes of continuous pumping.**

Hoveringham data set : change in resistivity

To show the change in resistivity more clearly, we take the difference in the logarithm of the model resistivity values.

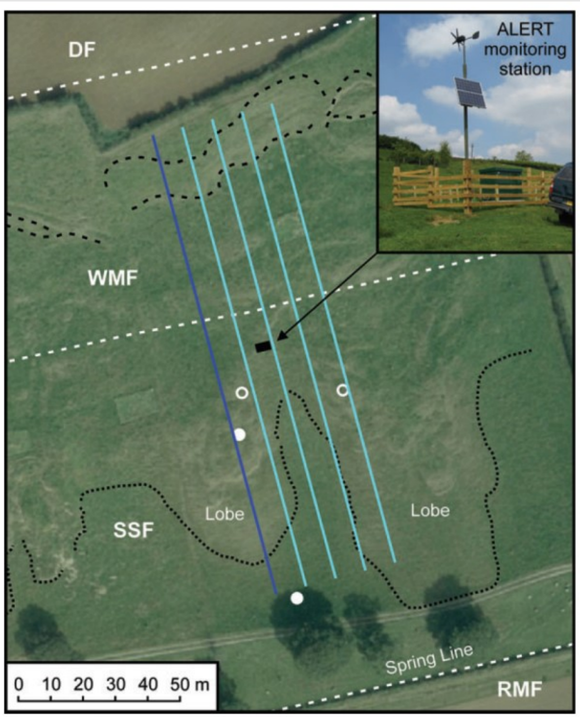
Sections showing the percentage relative change in the subsurface resistivity values with time obtained from the inversion of the data sets collected during the different stages of the pumping test.

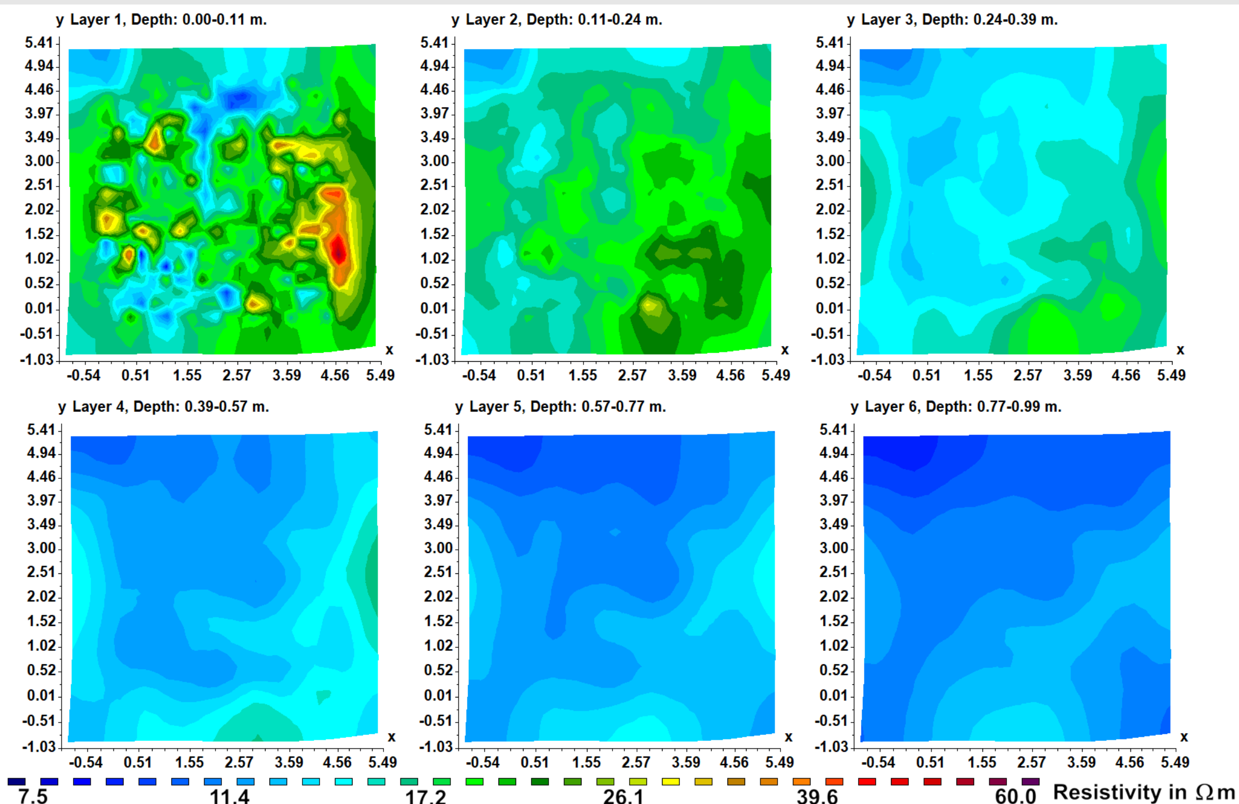
The increase in resistivity at depth due to the water extraction is clearly shown by the difference sections. The decrease near the top is due to water being poured out on the surface.



Field infiltration experiment, U.K.

An experiment to study the effect of fissures on hydrological processes was conducted at the Hollin Hill (U.K.) landslide site using electrodes in a 0.5 m grid. The landslide is on a slope that is 200 m long and 250 to 300 m wide with an average slope of 12°. The experiment was carried between 7th and 8th May 2014 by sprinkling 0.973 m^3 of salt water over a plot with an area of ~20 m^2. The right figure shows the resistivity inverse model before the infiltration as layers. The top drier layer has resistivity of 20 to 80 Ω.m. The deeper zone below 0.5 m has lower values of less than 15 Ω.m.





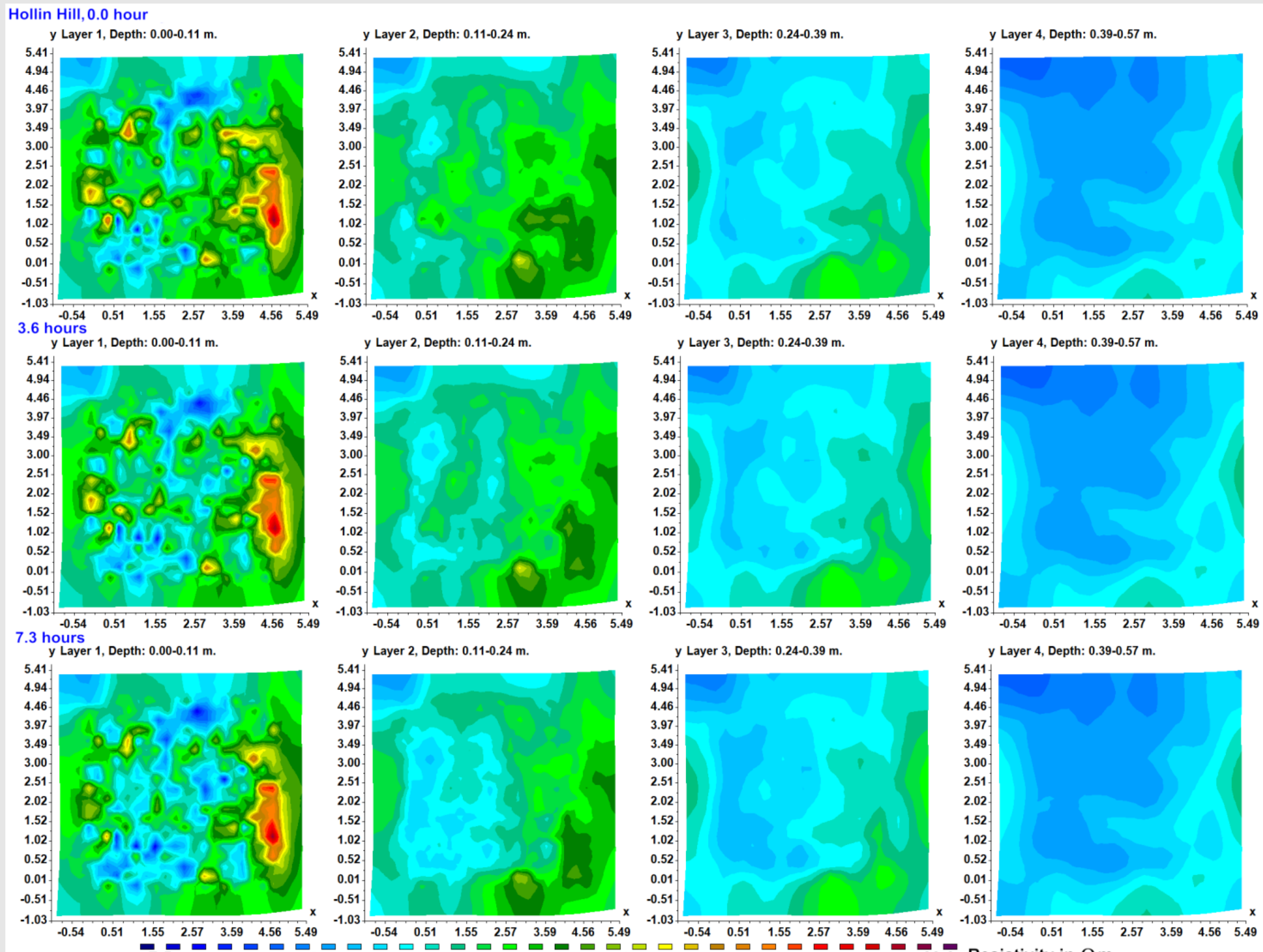
Field infiltration experiment, U.K.

The infiltration started 1.3 hours later lasting for 6.7 hours. 8 time-lapse data sets were collected. The models for the top 4 layers are shown at the 0.0, 3.6 and 7.3 hours. Some changes can be seen in the top 3 layers but the flow pattern is not clear due to the natural resistivity variations.

Before infiltration

2.3 hours after infiltration

6.0 hours after infiltration



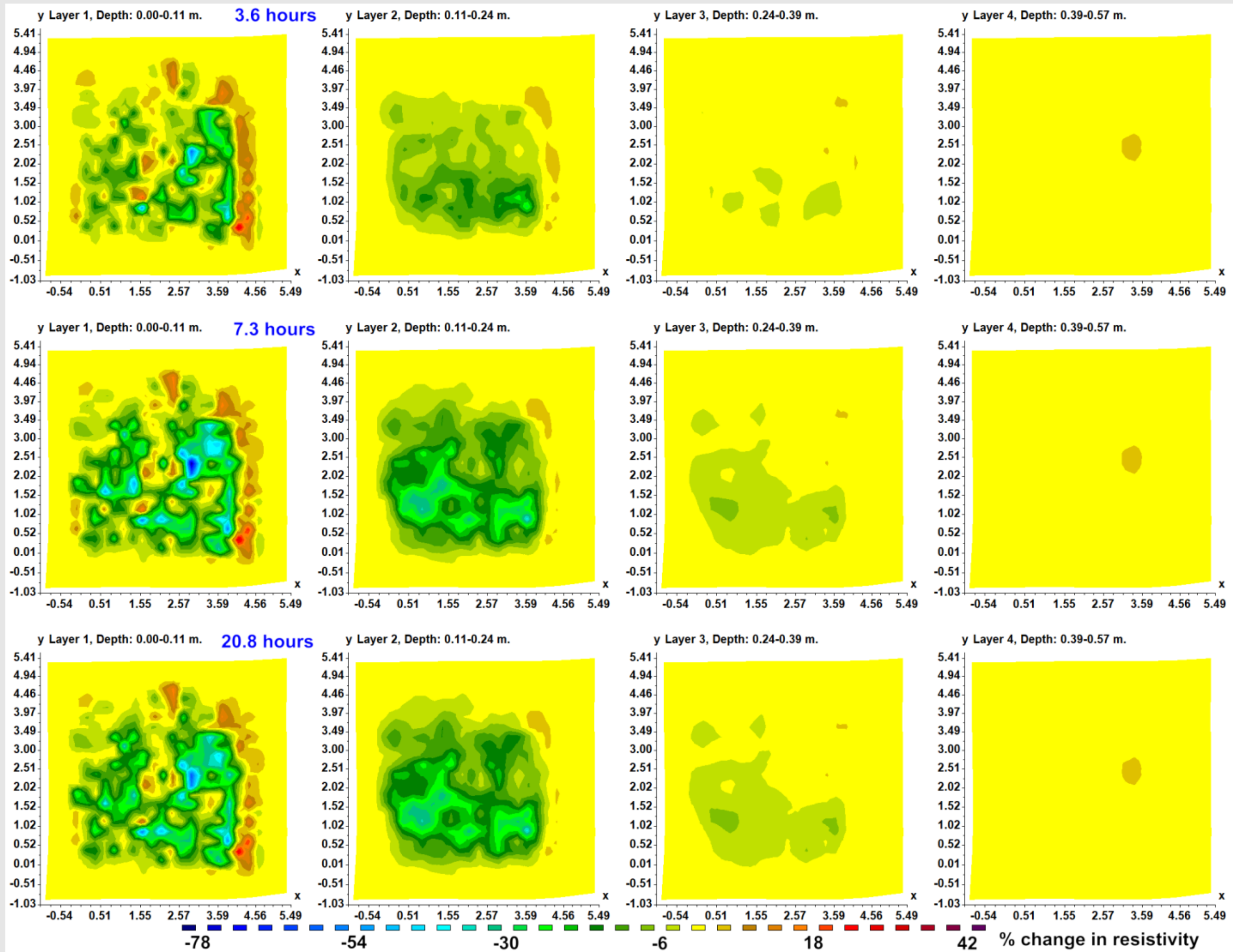
Infiltration experiment – difference sections.

The percentage change in the resistivity from the initial model are shown below.There is a large region with negative changes in the top 2 layers. However, there is band to the right of the infiltration zone in the top layer (near the shallow high resistivity zone) with increases of up to 30% that is an artifact.

2.3 hours after infiltration

6.0 hours after infiltration

12.8 hours after end of infiltration



A modified 4-D time-lapse inversion method

The standard 4-D least-squares method equation does not directly constrain the direction of the changes of the resistivity with time. The inversion algorithm is modified to remove the artifacts using 3 steps.

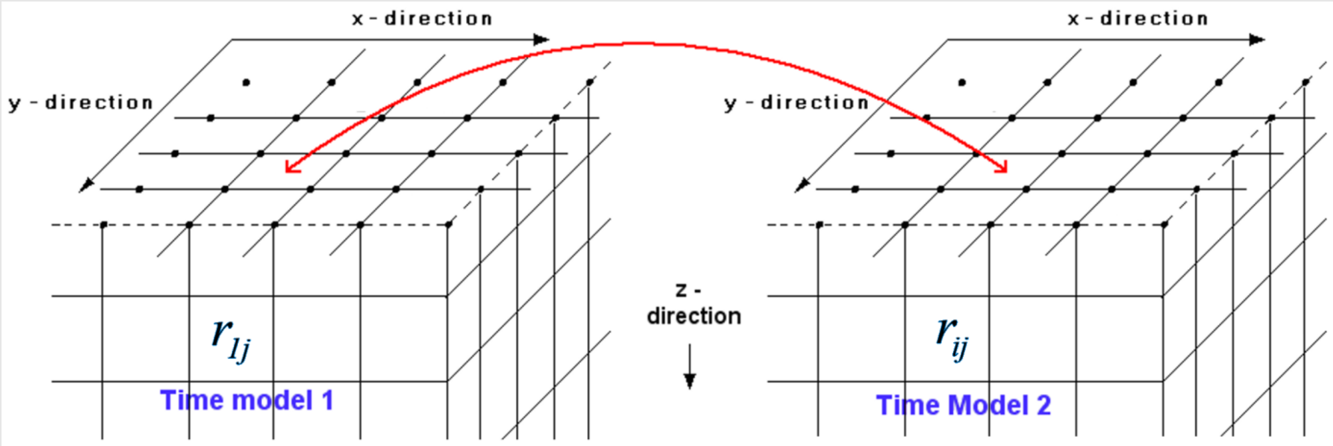
1). Carry out an **initial** inversion using the standard inversion algorithm.

2). Next compare the resistivity of each model cell in the first temporal model (r_{1j}) with the later temporal model (r_{ij}). Calculate the mean value $p_j=0.5*(r_{1j}+r_{ij})$ for each model cell.

3). If the model cell r_{1j} for the first temporal model is lower than p_j, it is reset to this mean value. If the same cell r_{ij} for the later temporal model is higher than p_j, it is reset to the same mean value (corresponding to zero change with time). This is the '**truncation**' step.

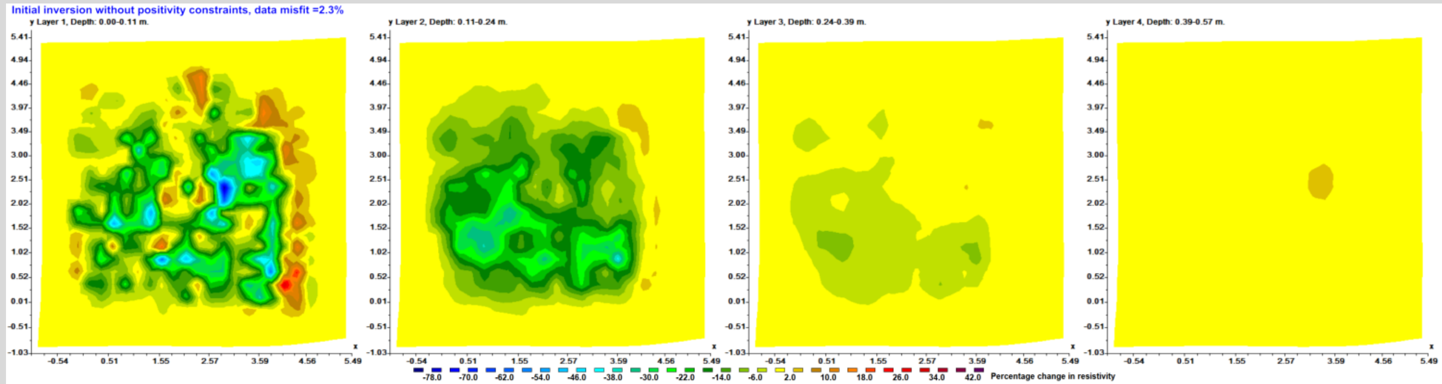
4). A **model refinement** inversion step is used with the transformed variables s_{kj}, where $r_{1j} = p_j + s_{1j}^2$, $r_{ij} = p_j - s_{ij}^2$

This transformation ensures the first model resistivity r_{1j} is always higher than the later model resistivity r_{ij}.

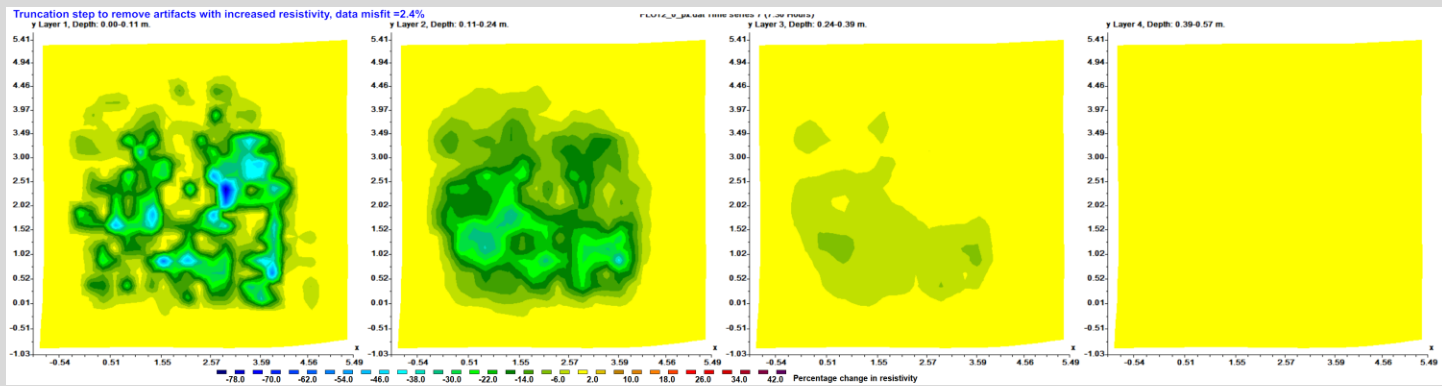


Example of positivity constrained inversion method

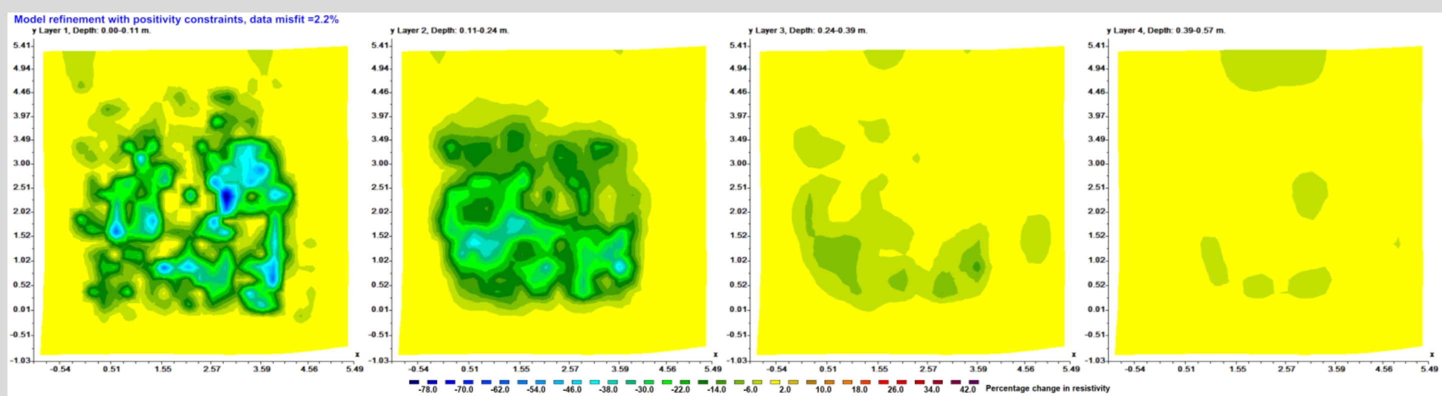
Initial model with standard 4-D inversion method (20.8 hrs data set). Artifacts in first layer.



Truncation step to remove artifacts. Slight increase in data misfit from 2.3% to 2.4%.



Refinement step with positivity constraints. Reduced data misfit to 2.2%.



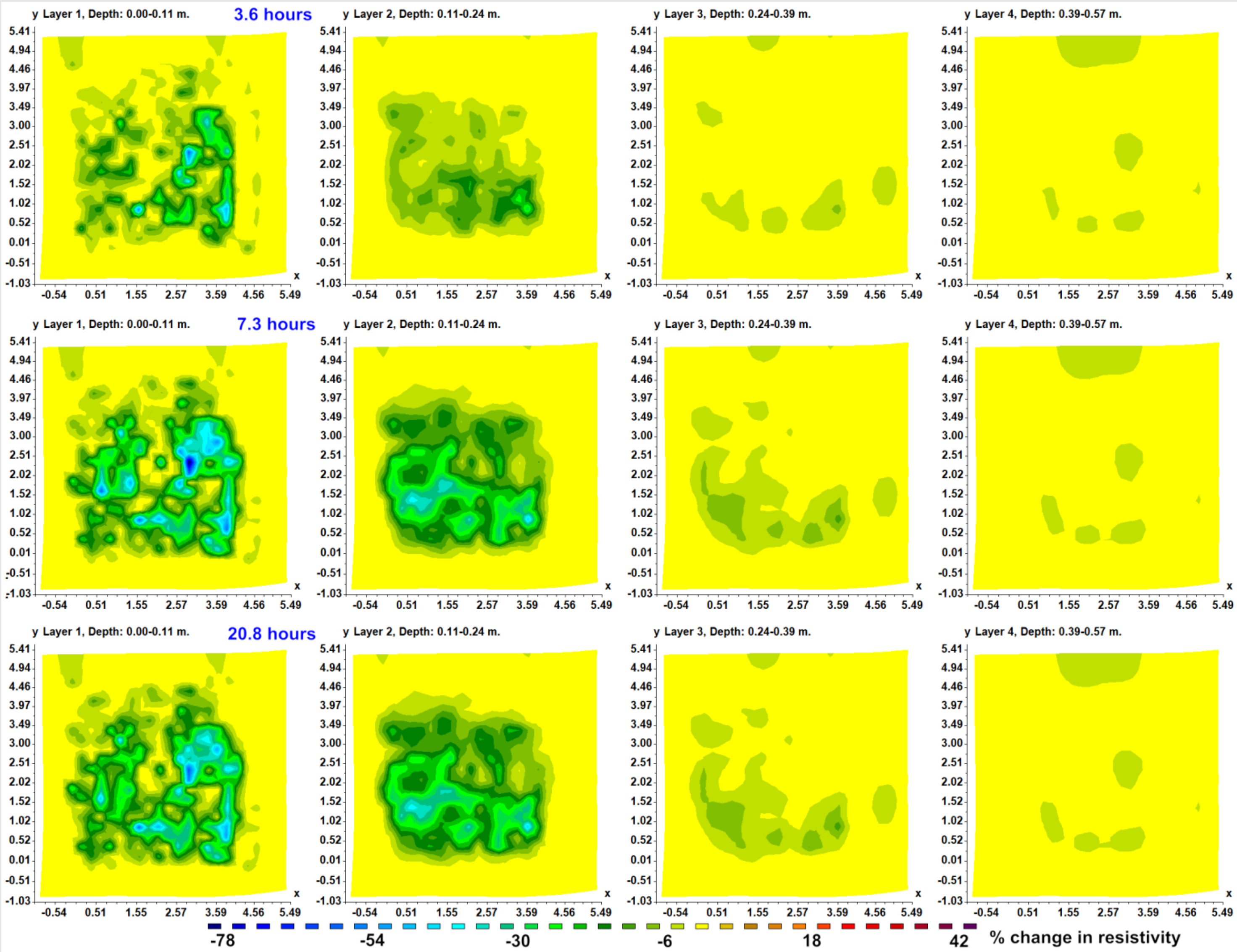
Infiltration experiment – modified difference sections

The artifacts are eliminated in difference sections with the positivity constraints. There is a slight reduction in the negative changes in the top layer from 7.3 to 20.8 hrs which is about 12.8 hrs after the infiltration was stopped.

2.3 hrs after infiltration

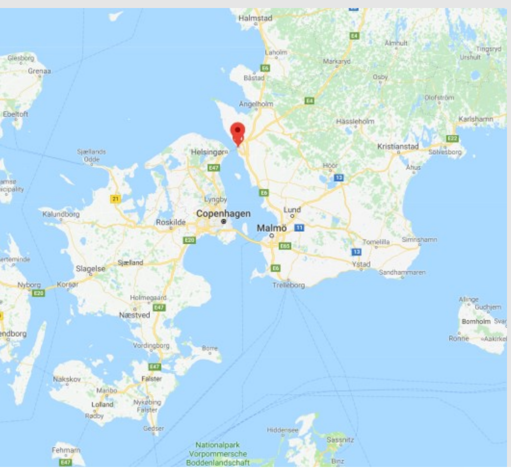
6.0 hrs after infiltration

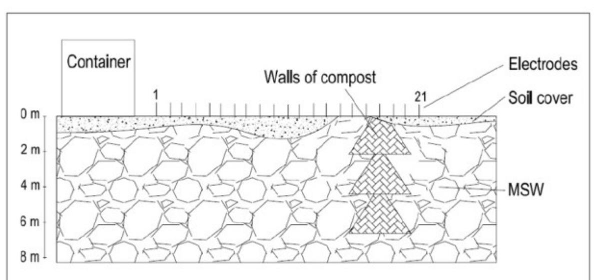
12.8 hrs after end of infiltration. Water seepage.

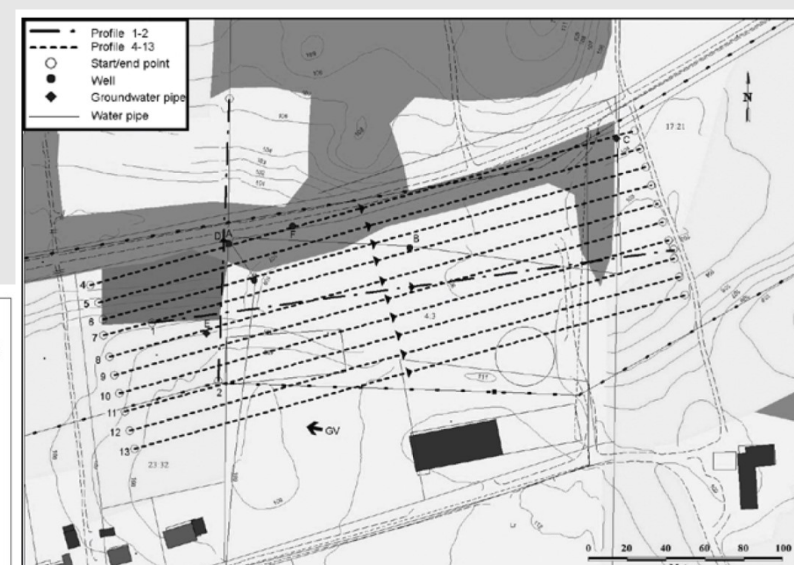


A more complex case : Landfill survey, Sweden

This survey was carried out to map methane gas accumulation in a landfill site in the Helsingborg area, Sweden. There is cover material consisting of soil about 1 to 2 m thick under which there is waste. The groundwater level is at about 5 m depth. There were 9 parallel 2-D survey lines with 21 electrodes with a spacing of 1 m using the pole-dipole array. The spacing between the lines was 2 m. Eight sets of measurements were made at 3 hrs apart for each day from 2008 to 2011 using an automatic monitoring system.

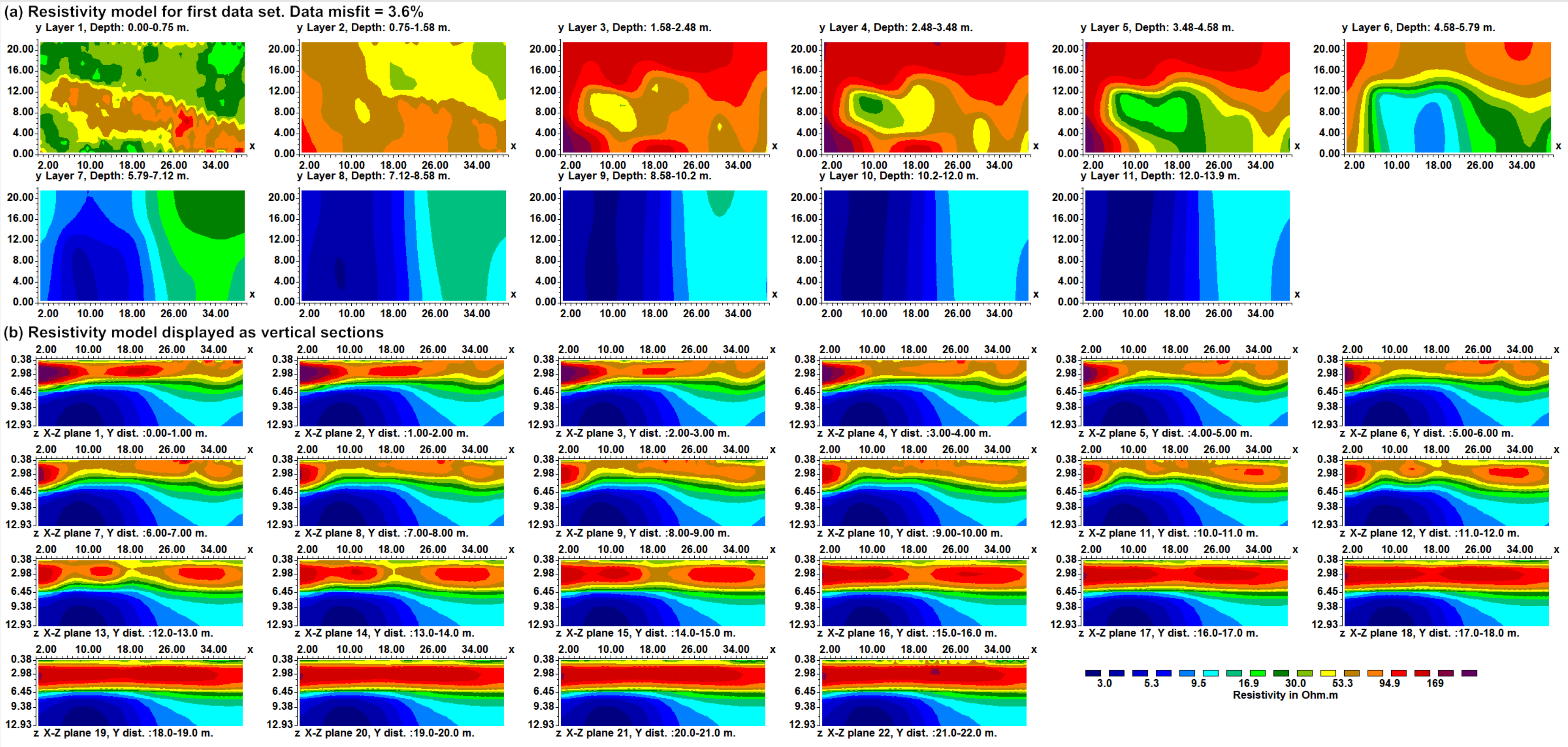






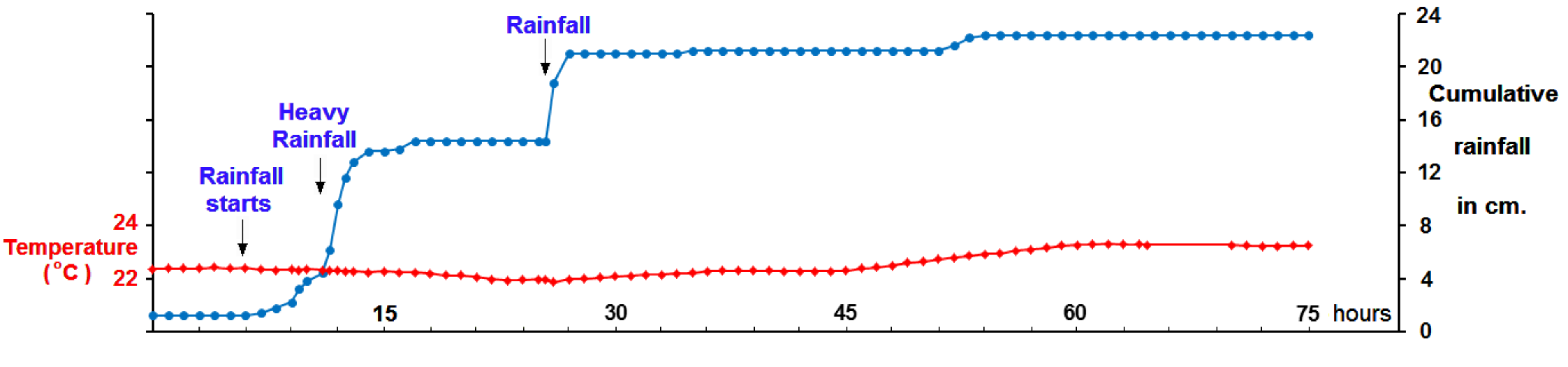
Landfill survey resistivity sections and stratification

The inverse model shows a thin top layer with resistivity values of 20 to 50 Ω.m and thickness of about 1 m which is the soil cover. There is a high resistivity linear band which might correspond to soil material with less clay which is more permeable than the surrounding soil, below which there is a band of lower resistivity values. Between 1 to 5 m depth is the landfill waste material above the water table with resistivity values of between 50 to 400 Ω.m. The water table at about 5 m depth is marked by a sharp decrease in the resistivity values to below 10 Ω.m.



Landfill survey, rainfall record

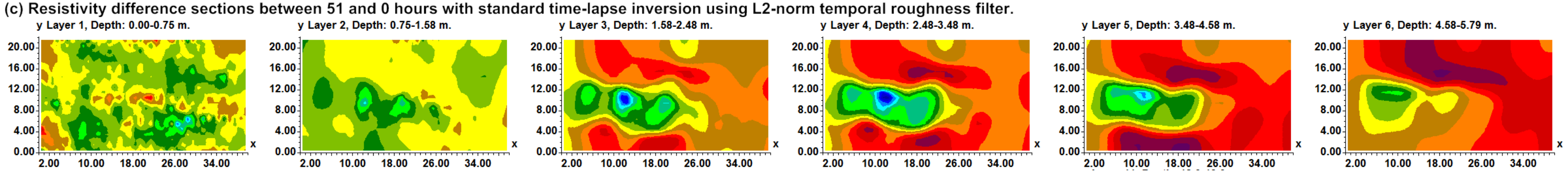
On 2nd July 2011 there was heavy rainfall that cause significant changes in the resistivity. There is slight rainfall starting at about 6.00 hrs with the heaviest rainfall between 9 to 15 hrs. A second rainfall event occurred between 27 and 28 hrs, and slight rainfall at about 51 hrs. The temperature from a probe at 0.3 m depth showed slight variations of about 1°C which will not cause significant changes in the resistivity. A time-lapse inversion of 26 data sets measured 3 hrs apart was carried out to map the resistivity changes due to migration of the water from the rainfall. We first show the changes in the resistivity between the inverse models at 0 and 51 (after the last rainfall) hrs.



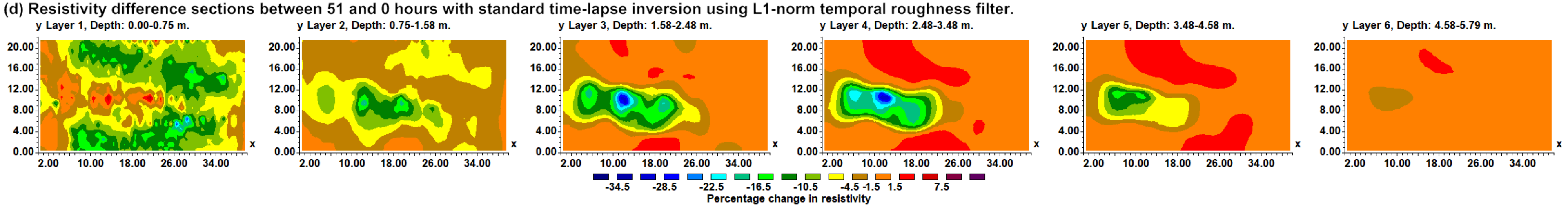
Landfill survey, standard 4-D time-lapse inversions

Figure below shows the percentage resistivity change between the models at 51 and 0 hrs obtained by the inversion algorithm using the L2 and L1 norm time roughness filter. Both models show a large zone with reduction in resistivity of up to 30% between 1.6 to 4.6 m depth due to the rainwater that has migrated downwards through the more permeable region. There were no changes after the 6th layer below the water table. There are small regions in the surface high resistivity linear band with increases in resistivity of up to 8%. The model obtained using the L2-norm filter shows increases of over 10% in the 3rd to 6th layers at the sides of the low resistivity zone. The increase in resistivity is reduced to about 4% when the L1-norm filter is used. The zones with an increase in resistivity in the deeper layers are likely to be artifacts as they are highly dependent on the constraints used.

(c) Resistivity difference sections between 51 and 0 hours with standard time-lapse inversion using L2-norm temporal roughness filter.

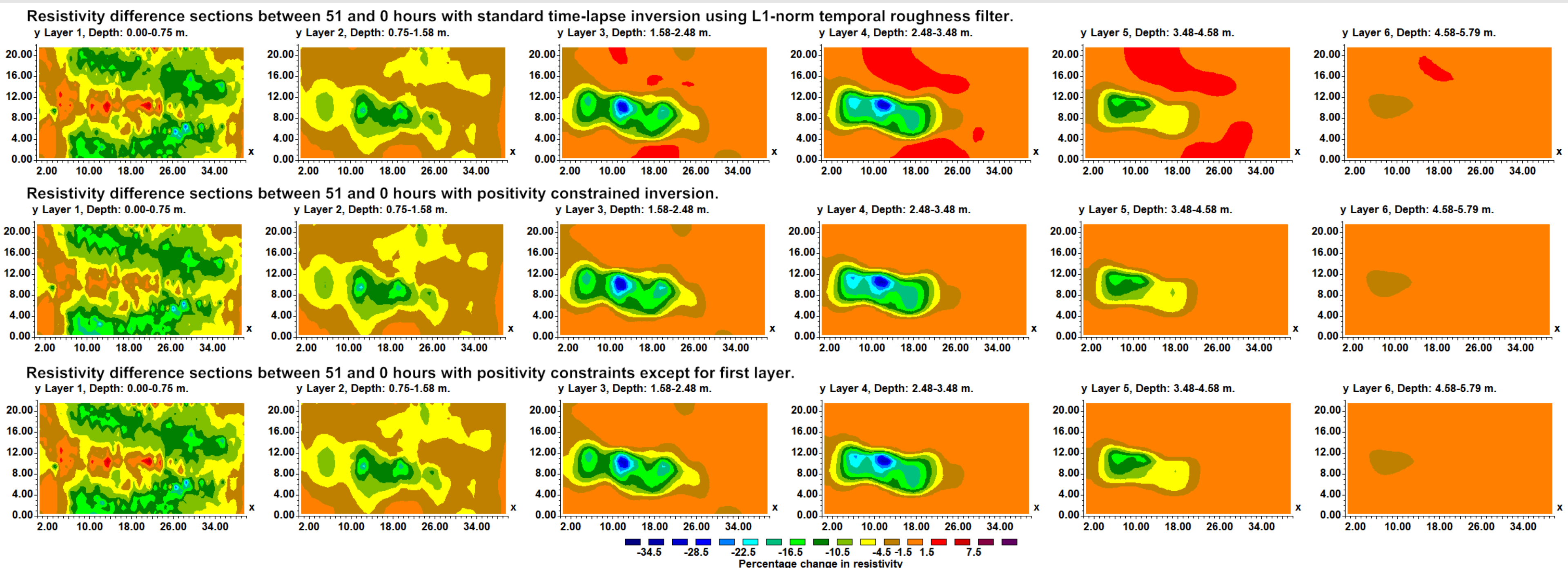


(d) Resistivity difference sections between 51 and 0 hours with standard time-lapse inversion using L1-norm temporal roughness filter.



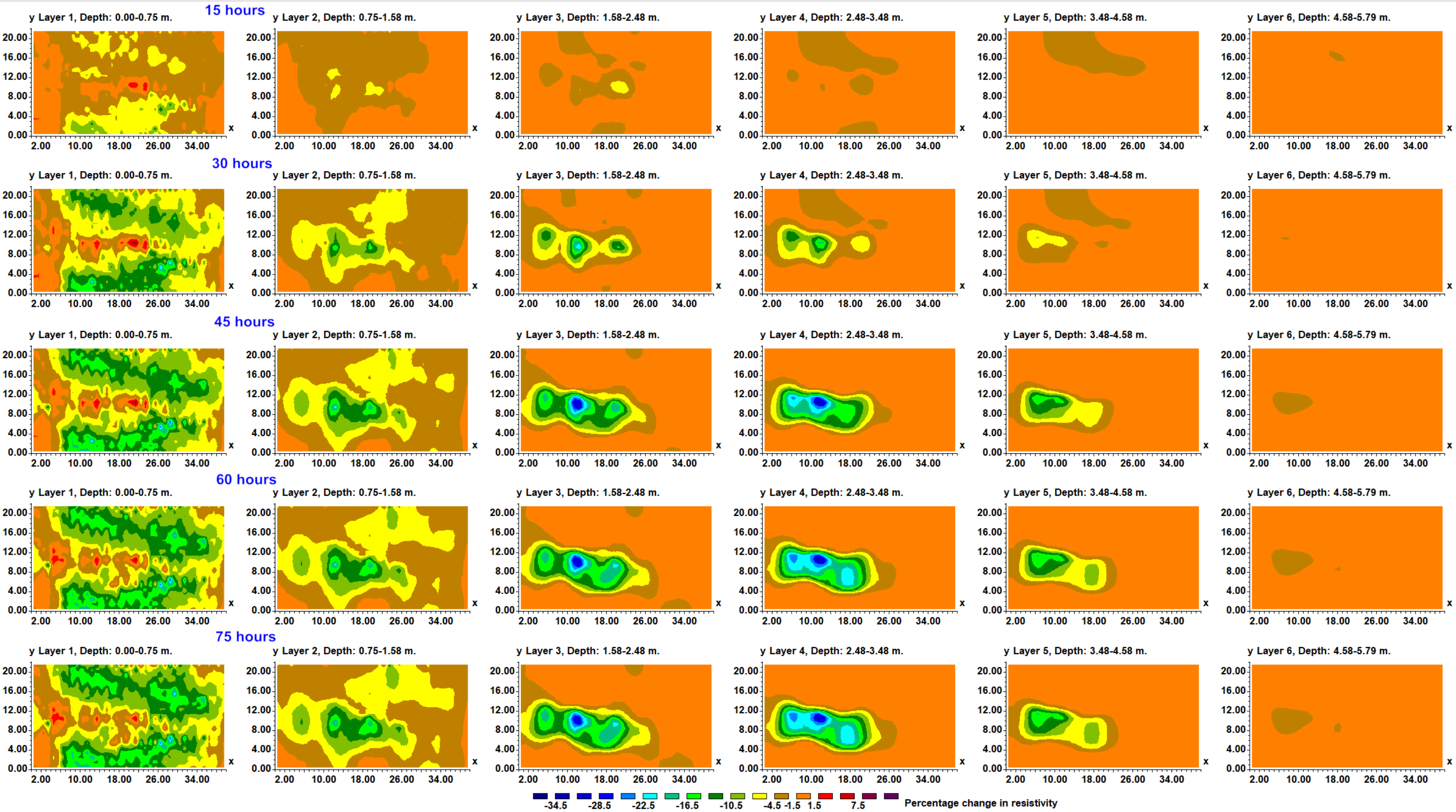
Landfill survey difference sections with positivity constraints

The 2nd row shows the difference sections obtained after applying the positivity constraints. The zones with an increase in resistivity have been removed in all the layers. Some studies indicate that the increase in resistivity in the top layer might be real, being caused by gas accumulation below soil due to water saturated pores hindering the release of gas to the air. The 3rd row shows the inverse model obtained when the positivity constraints are not applied to the top layer. The resulting model allows resistivity increases in the top layer while removing the artifacts in the deeper layers. All the models have the same data misfit of 3.6%.



Landfill survey, change of difference sections with time

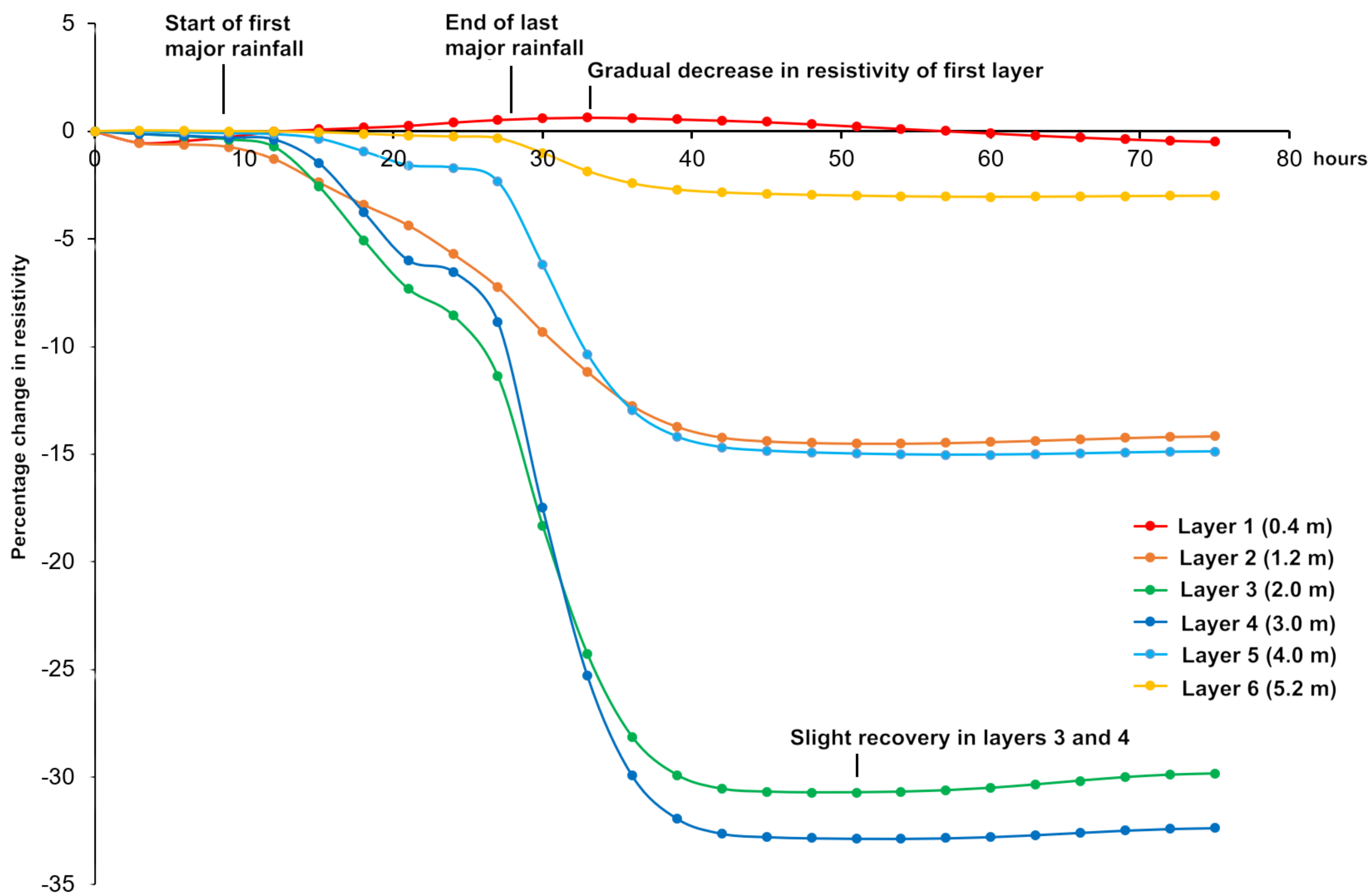
The difference sections at every 15 hrs, from 15 to 75 hrs, are shown. They show the slow migration of the water from the surface to the deeper layers, particularly in the first 45 hrs.



Landfill survey, plot of resistivity changes with time

Plot of the change at a cell located at (11.5m,10.5m) within the permeable zone in the different layers. The first layer shows a slight increase in the resistivity due to gas accumulation in the saturated soil. The resistivity of layer 2 drops before layer 3 which drops before layer 4. There is a slight recovery in layers 3 and 4 some time after the last major rainfall at 28 hours due to drainage of the water downwards to the water table.

The resistivity of the top layer decreases after 32 hrs due to release of gas to the air.



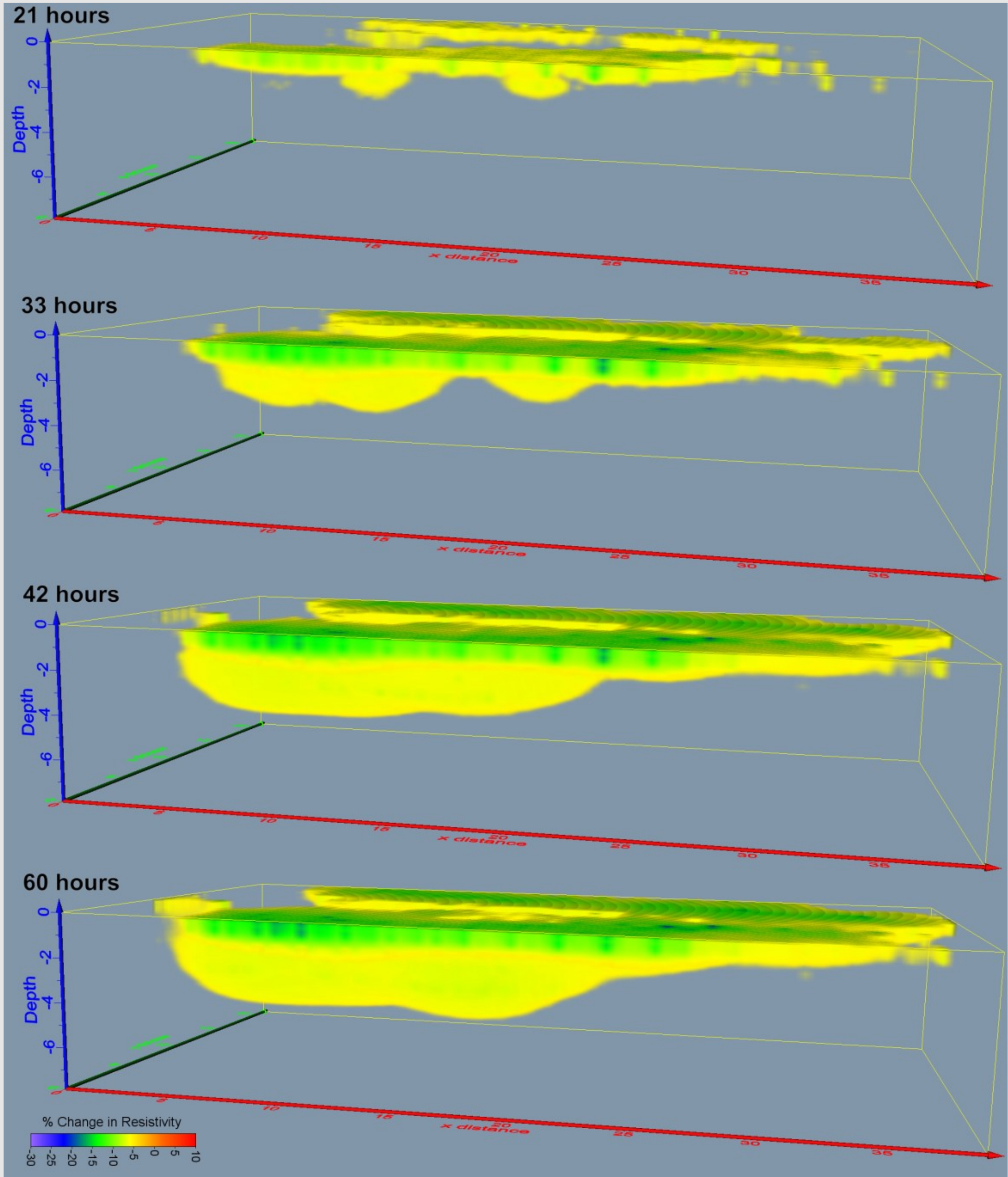
Landfill survey, 3-D side plots of resistivity change

We show 3-D plots of the change in the resistivity between 21 and 60 hrs, as viwed from the side.

The plot shows regions where the resistivity had decreased by between 5 and 30 %.

There is a steady migration of the low resistivity plume downwards with time within a narrow permable zone.

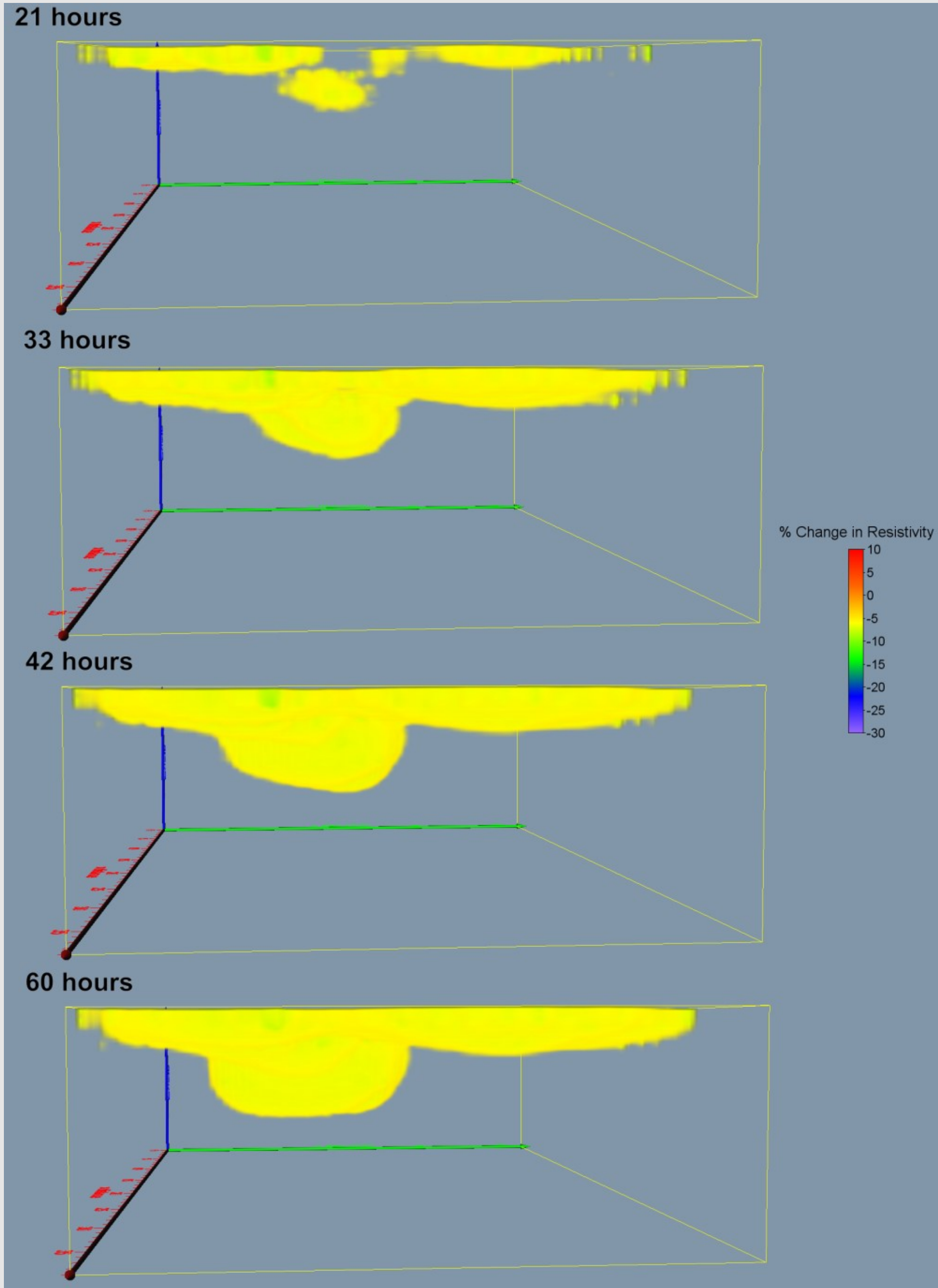
By about 60 hrs it had reached its maximum extent.



Landfill survey, 3-D front plots of resistivity change

This shows 3-D plots of the change in the resistivity between 21 and 60 hrs, as viewed from the front.

The limited width of the permeable zone where the water has migrated downwards is more clearly shown.



Conclusions

While the standard 4-D time-lapse inversion algorithm generally produces acceptable results, it does not explicitly constrain the direction of the changes with time. In some situations with large resistivity variations (both spatially and temporally), it can produce artifacts where the inverse model resistivity increases when it is expected to only decrease (or vice versa).

These artifacts can be removed by using a modified 4-D inversion algorithm. The model from the standard 4-D inversion algorithm is used as a starting model. The resistivity values of the different time models are then modified in a truncation step to remove the artifacts. It is followed by a refinement step where the method of transformations is used to ensure that the changes in time in the inverse models only occur in the expected direction.

The constraints can be modified so that they are only applied to selected regions in the model. This allows increases in selected parts of the model (based on independent information) while removing artifacts in other regions.