Methods to reduce banding effects in 3-D resistivity inversion

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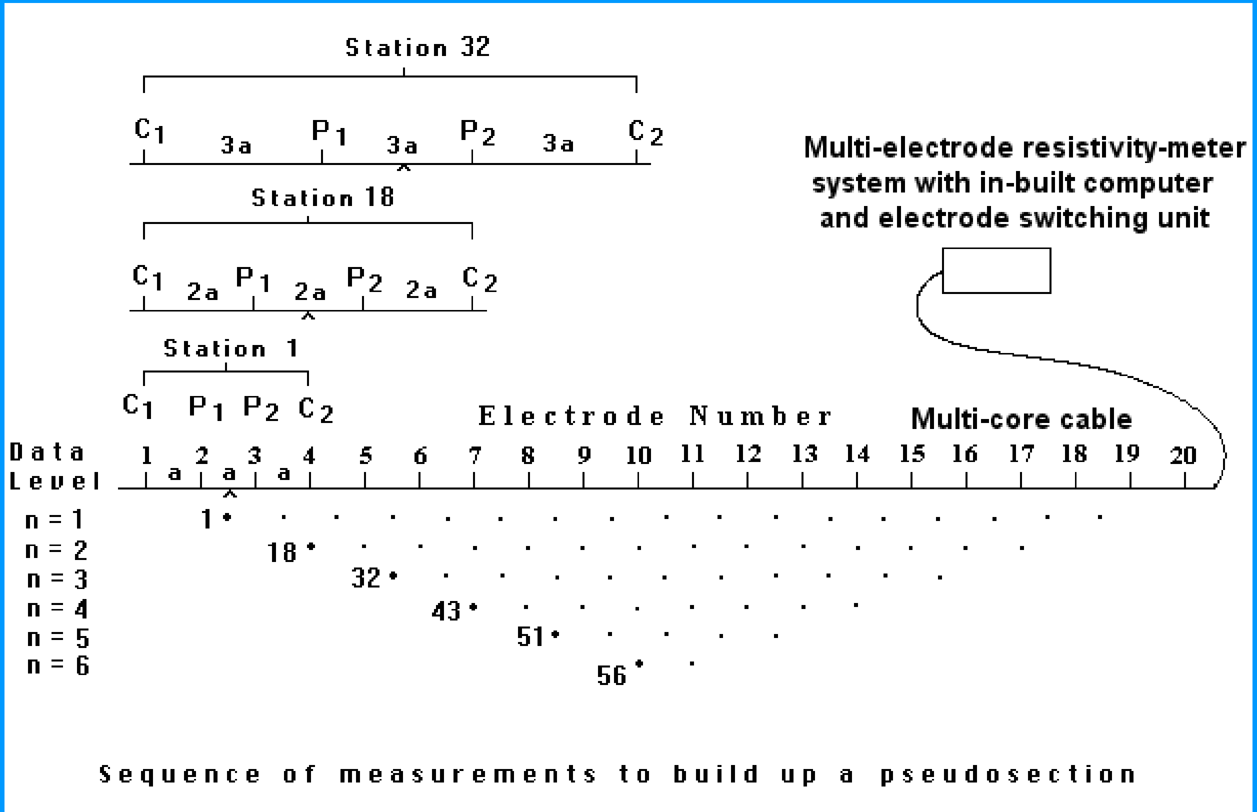
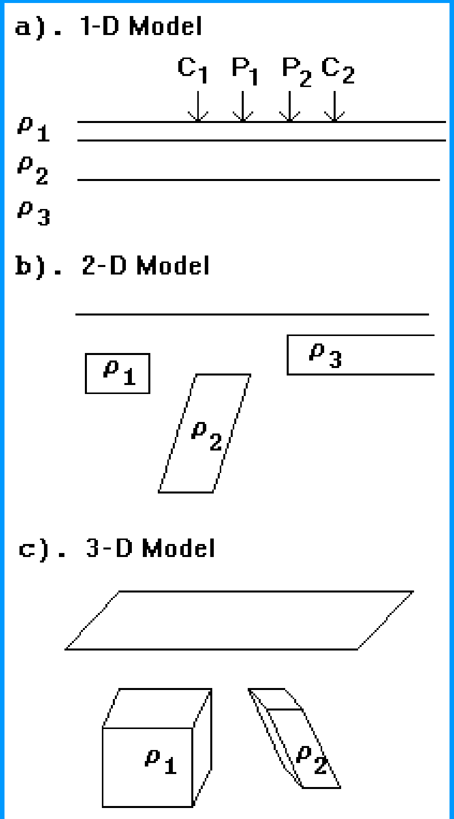
2-D electrical imaging surveys

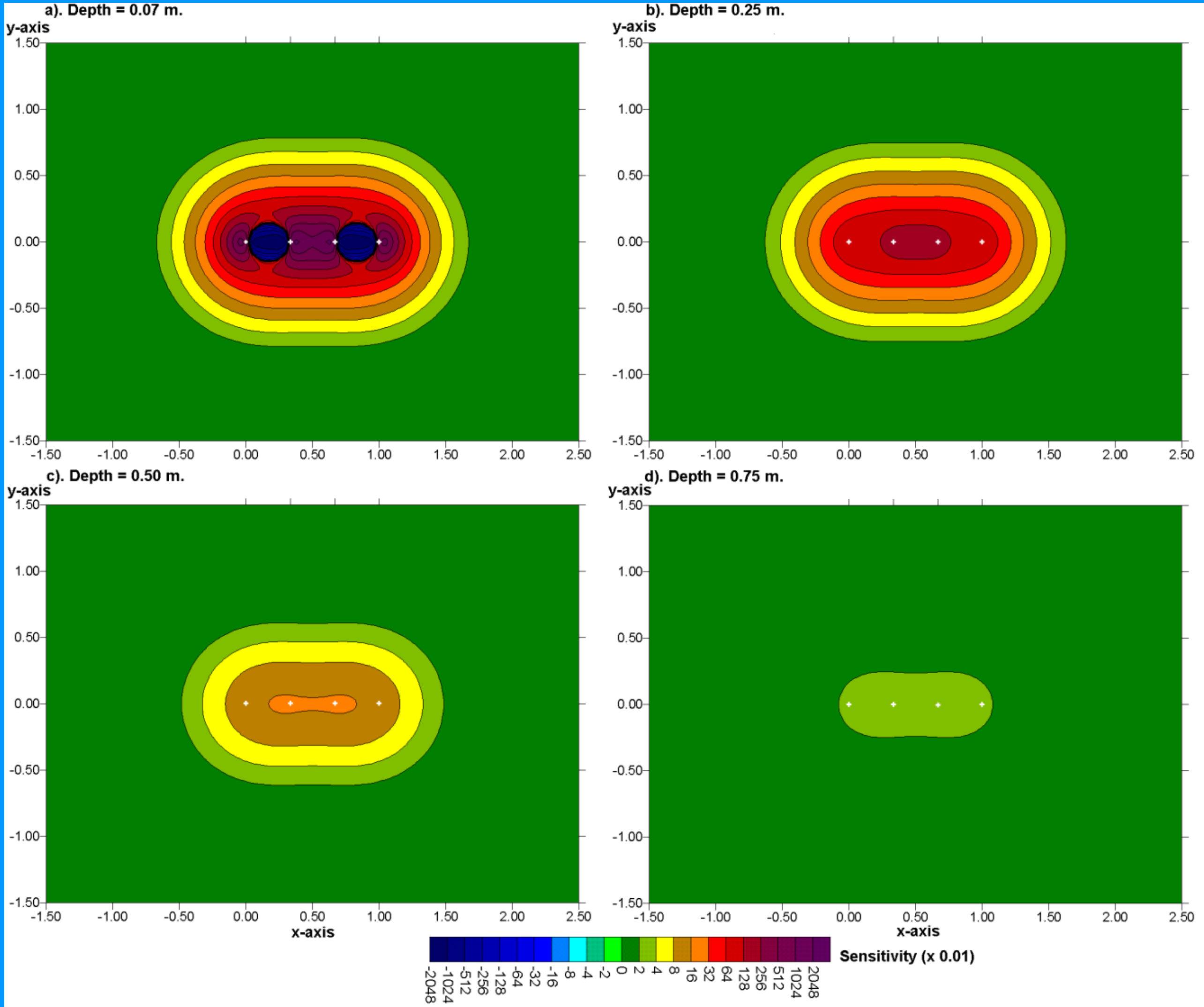
2-D electrical imaging surveys are now widely used. They give more realistic models than 1-D sounding surveys and can be efficiently carried out at a low cost. However, in very complex environments, a 2-D model might not be sufficiently accurate due to off-axis structures. A 2-D model assumes there in no change in the resistivity perpendicular to the survey line.





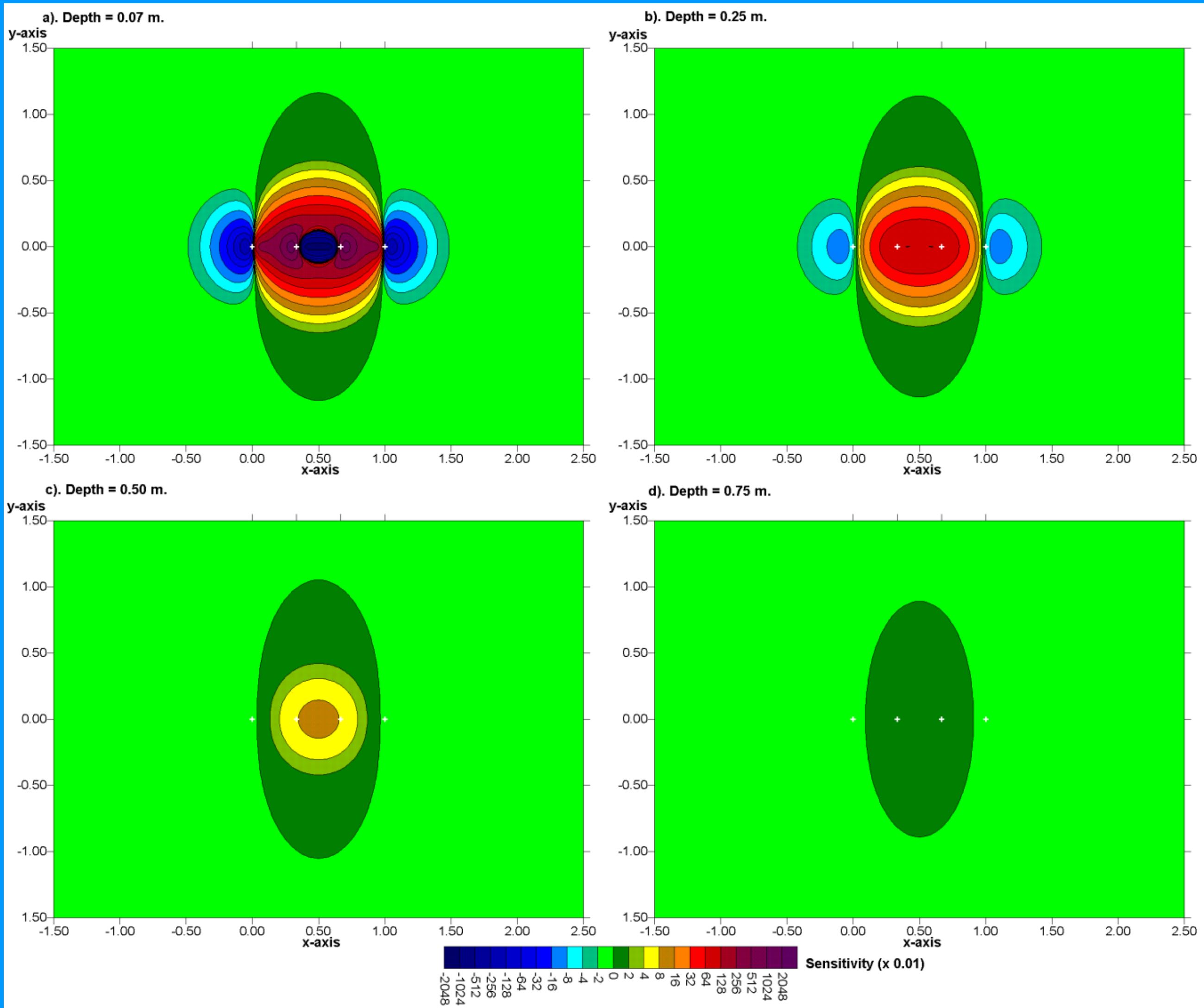
Source of 3-D effects

A linear electrode array is also affected by material away from its axis. Plots of sensitivity sections for the Wenner array at different depths show the sensitivity to off-axis structures. It is sensitive to structures up to about 1.5 to 2 times the ‘a’ electrode spacing.



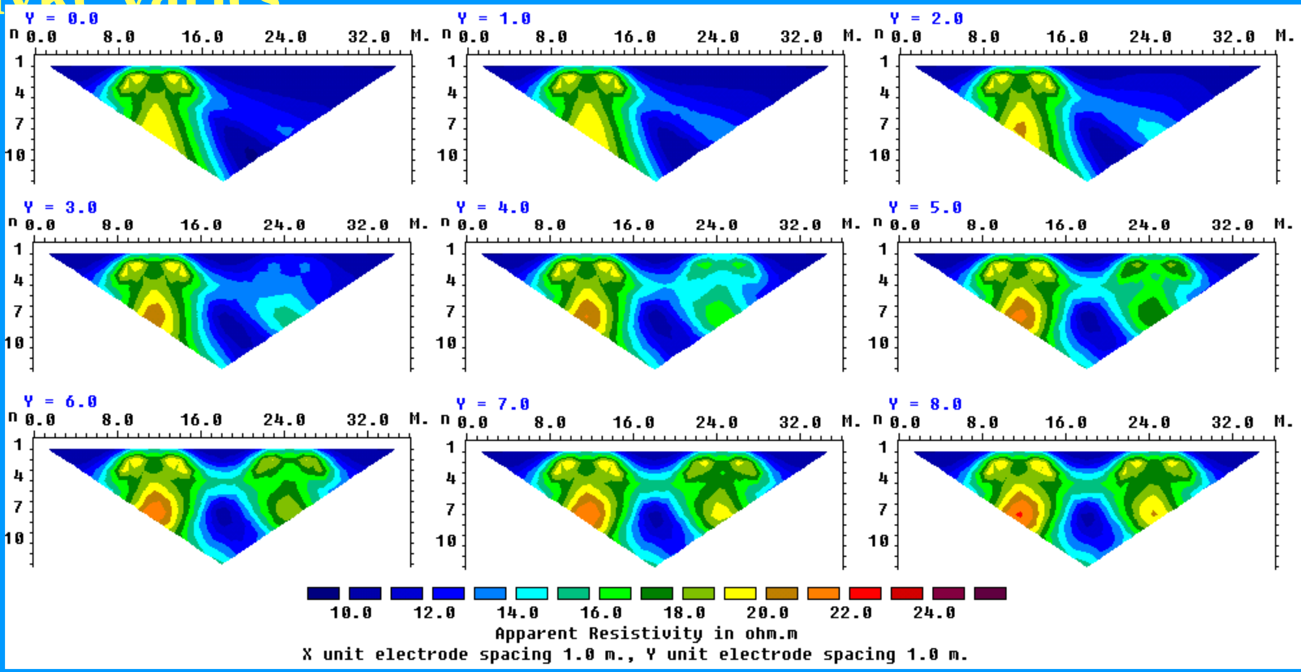
Dipole-dipole array 3-D effects

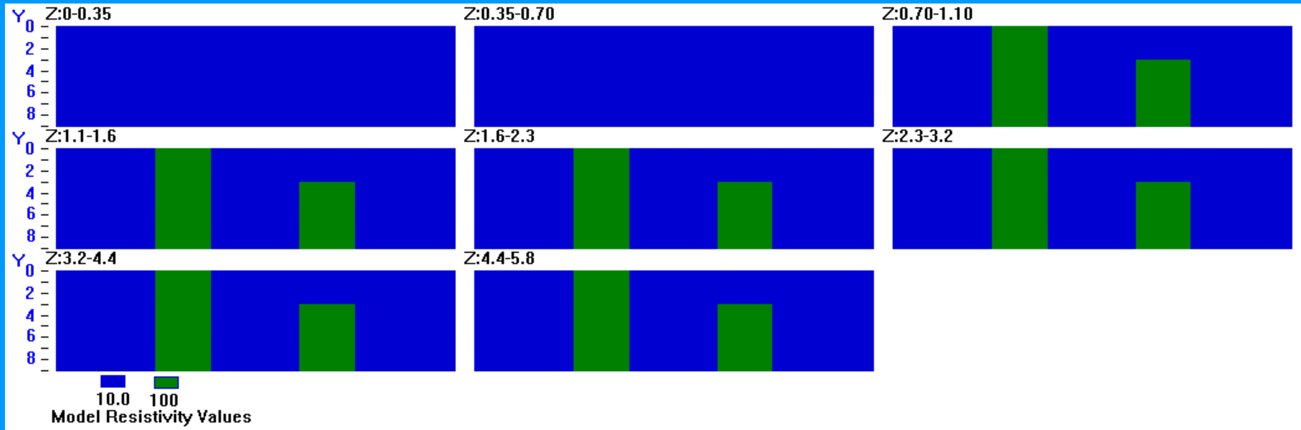
The dipole-dipole array is even more sensitive to 3-D effects. The sensitivity plot for dipole-dipole array with n=1 shows it is sensitive to structures up to about 2 to 3 times the ‘a’ spacing. The sensitivity to off-axis structures increase with the ‘n’ value.



Example of anomalies from off-axis structure

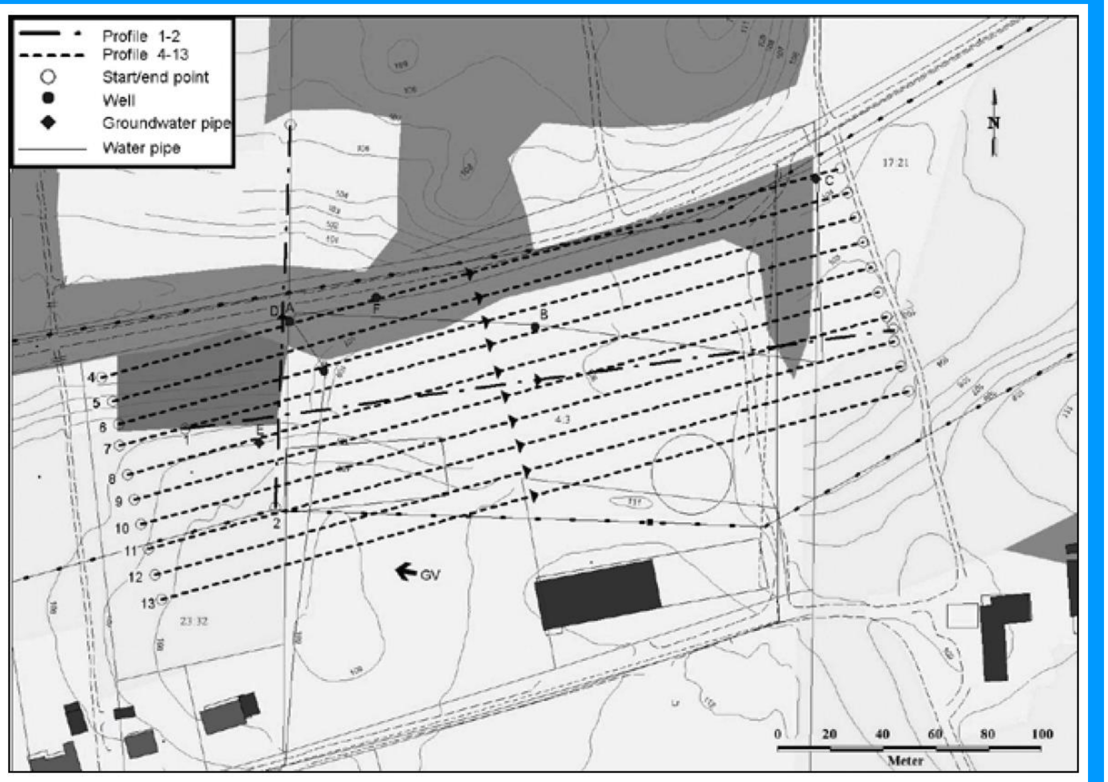
Example of Wenner array pseudosection from parallel survey lines over two high resistivity structures. The amplitude of the anomaly due to the truncated dyke varies with the distance from the edge. 2-D inversions of individual profiles will give a dyke structure with varying depth, width and resistivity.

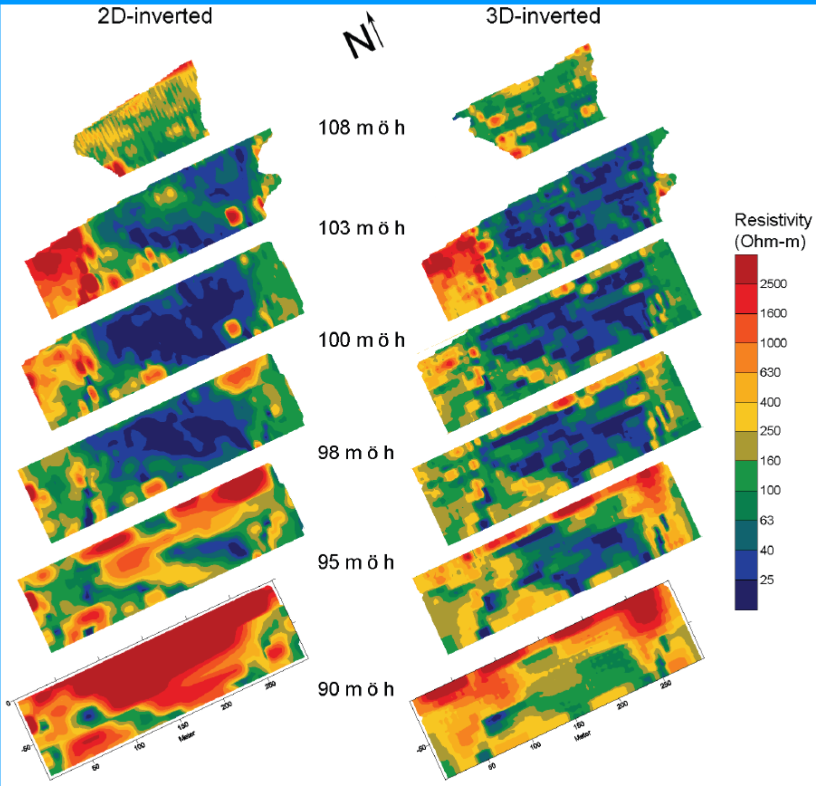




Field example of 3-D effects

Survey over the Ekeboda landfill with parallel 2-D lines. The low resistivity area in the 2-D models due to leachate downward migration at 95 m. elevation is much smaller than the 3-D model. Other known structures such as a buried culvert on the eastern side at 98 m. elevation shows up better in the 3-D model. However the 3-D model shows prominent artefacts that are aligned along (or perpendicular to) the direction of the survey lines.

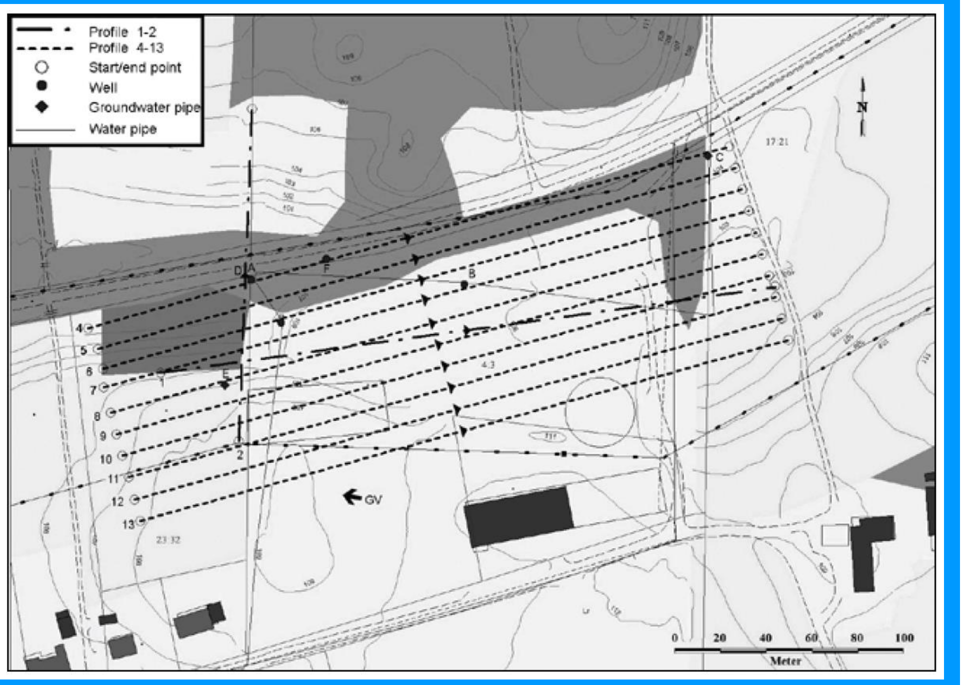


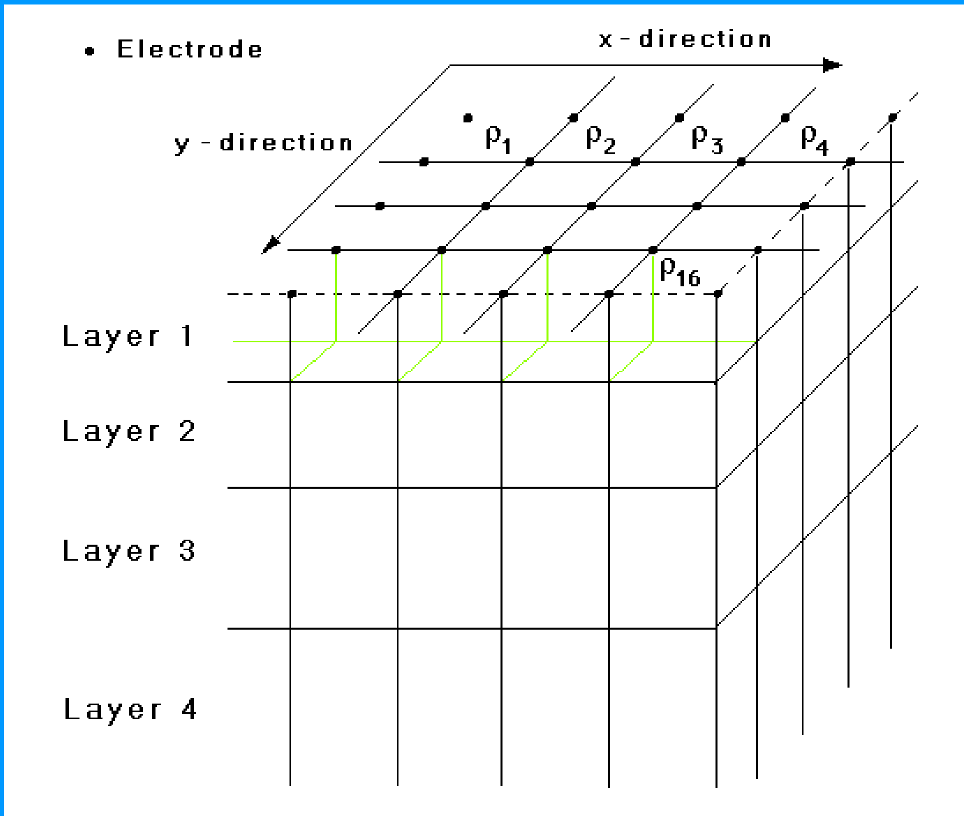


Reasons for artefacts in 3-D inversion model

The artefacts are due to the survey setup, the arrangement of model cells and the smoothness-constrained least-squares method used. The measurements are usually made in only one direction.

The x (and y axis) of the model cells is arranged along (and perpendicular to) the direction of the survey lines. There is a directional bias in both the data and the model cells setup.



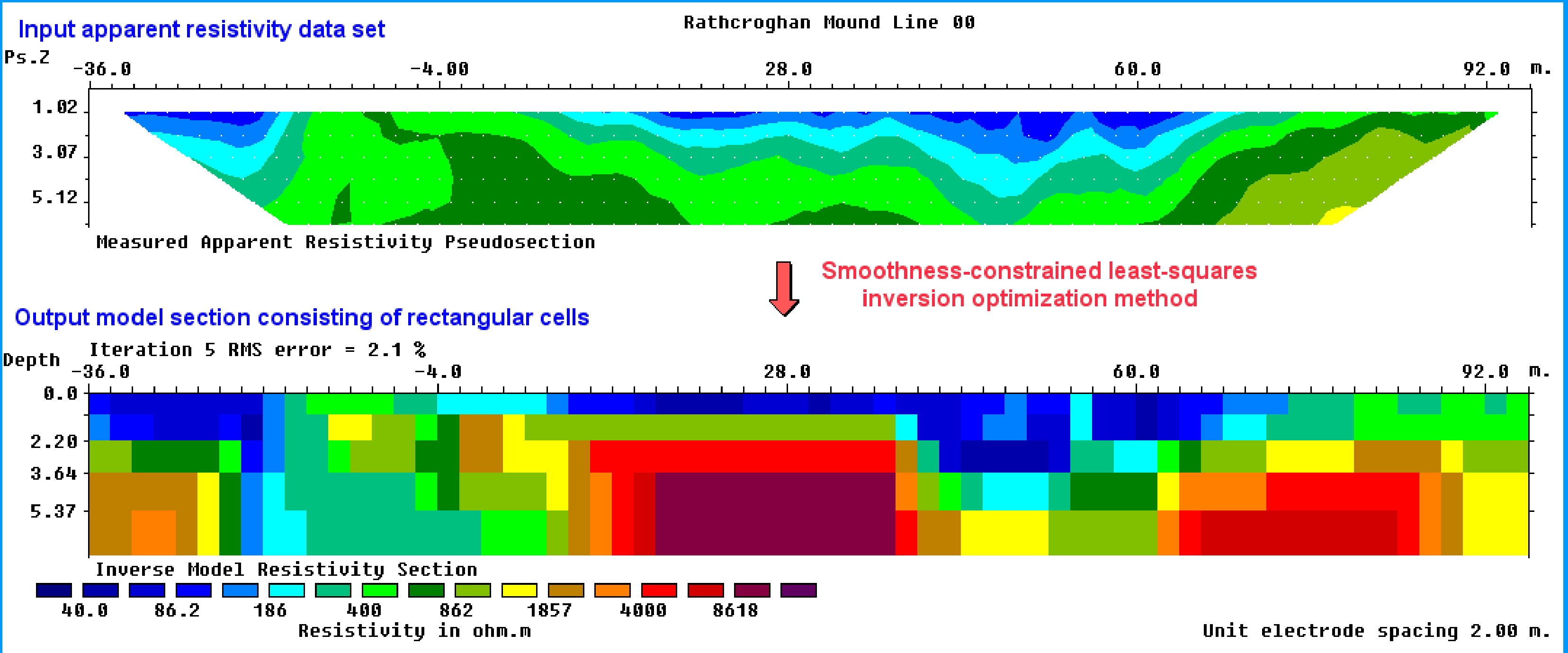


Smoothness constrained least-squares method

The equation for the smoothness-constrained least-squares inversion method equation is given by

$$\left(\mathbf{J}^{T}\mathbf{J} + \lambda\mathbf{F}\right)\Delta\mathbf{q}_{k} = \mathbf{J}^{T}\mathbf{g} - \lambda\mathbf{F}\mathbf{q}_{k-1}$$

F contains the roughness filters and other constraints that stabilizes the inversion, λ is a damping factor, Δq is the change in model resistivity values, and g is the data misfit. J is the Jacobian matrix of partial derivatives.

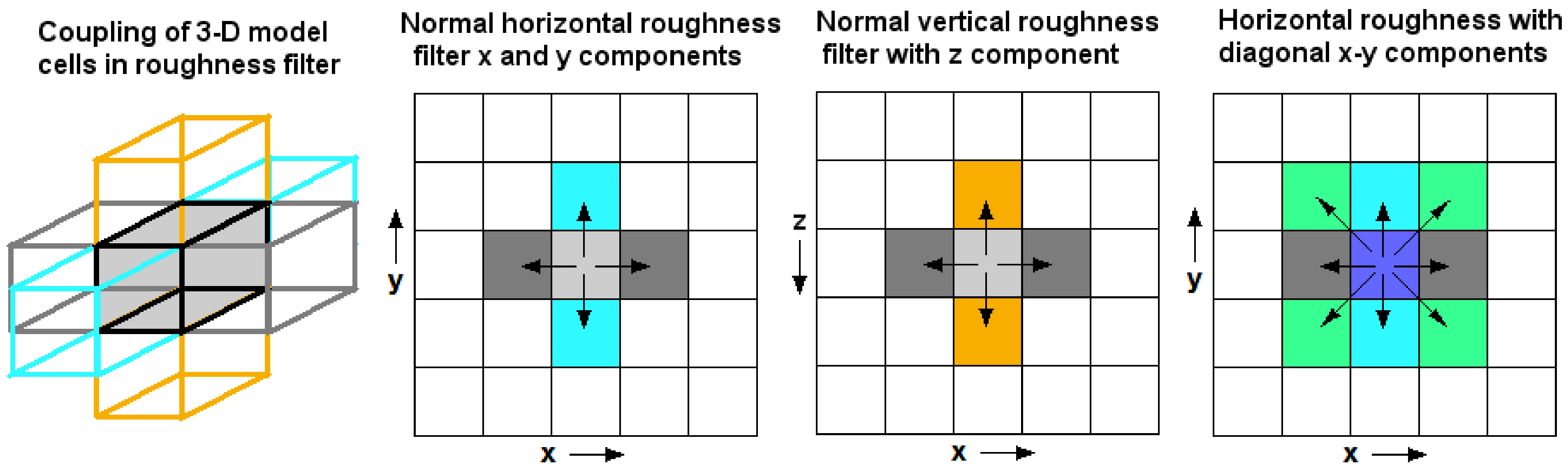


Structure of the roughness filter

The roughness filter has the form $$F = \delta x^T \delta x + \delta y^T \delta y + \delta z^T \delta z$$

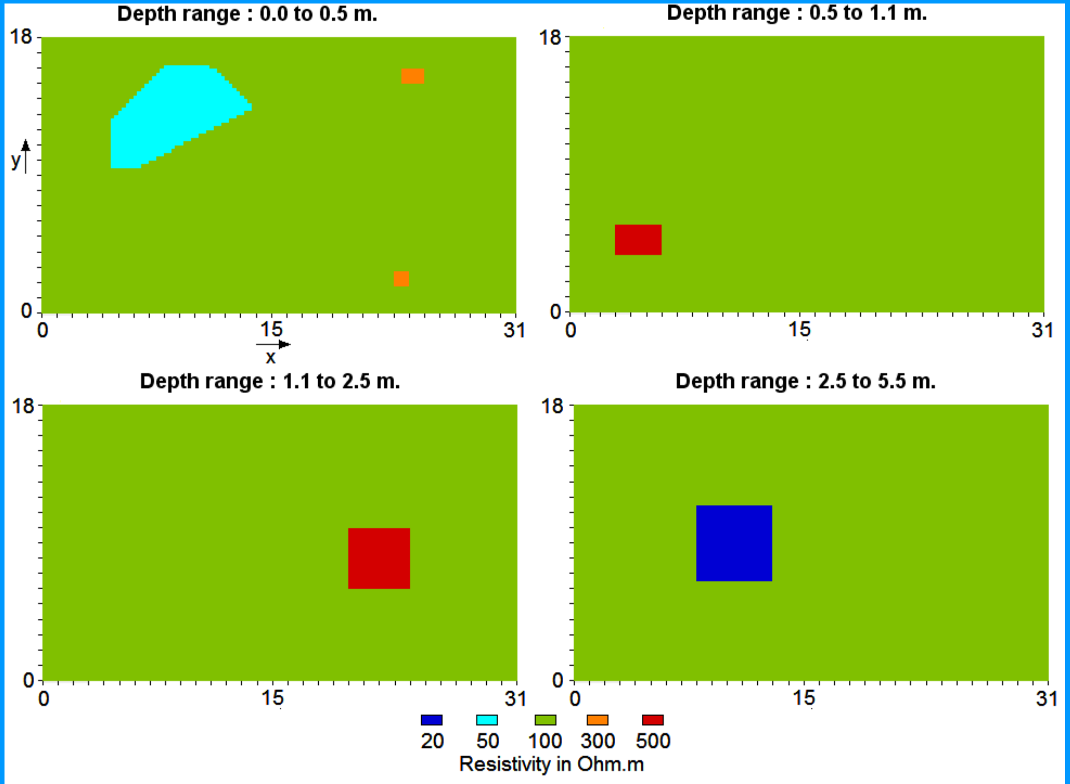
δx, δy and δz are the first-order difference matrices in the x, y and z directions. It minimizes the change the resistivity between adjacent model cells in the x, y and z directions. It has a bias to produce structures that are aligned along the x, y and z directions particularly if the L1-norm (blocky) inversion method is used.

A modification to the horizontal roughness filter to include components in the diagonal x-y directions can be made to reduce the bias.



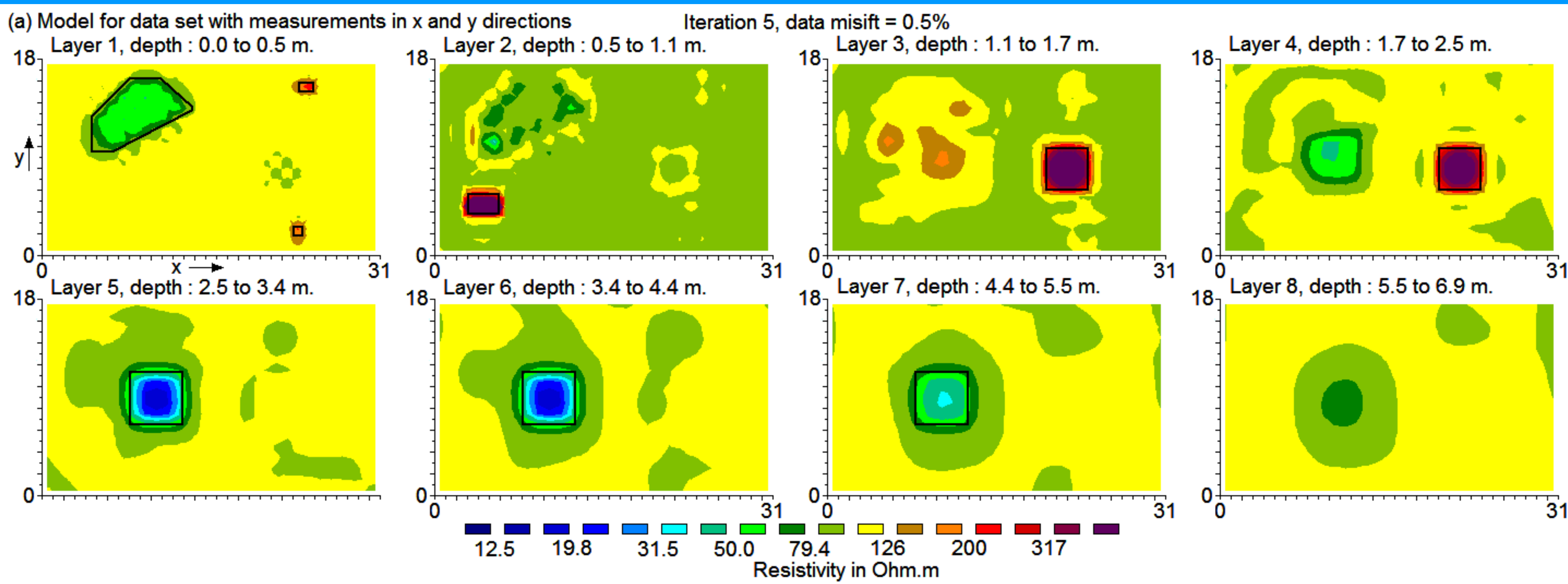
Synthetic test model

The test model has a broad low resistivity area (50 Ω.m) and two small high resistivity anomalies (200 Ω.m) in the top layer. There are two high resistivity (500 Ω.m) blocks at depth ranges of 0.5 to 1.1 and 1.1 to 2.5 meters. There is a large low resistivity (20 Ω.m) block at a depth range of 2.5 to 5.5 meters. All the anomalies are embedded in a medium of 100 Ω.m. The survey grid consists of 32 by 19 electrodes with a 1 meter spacing between the electrodes.



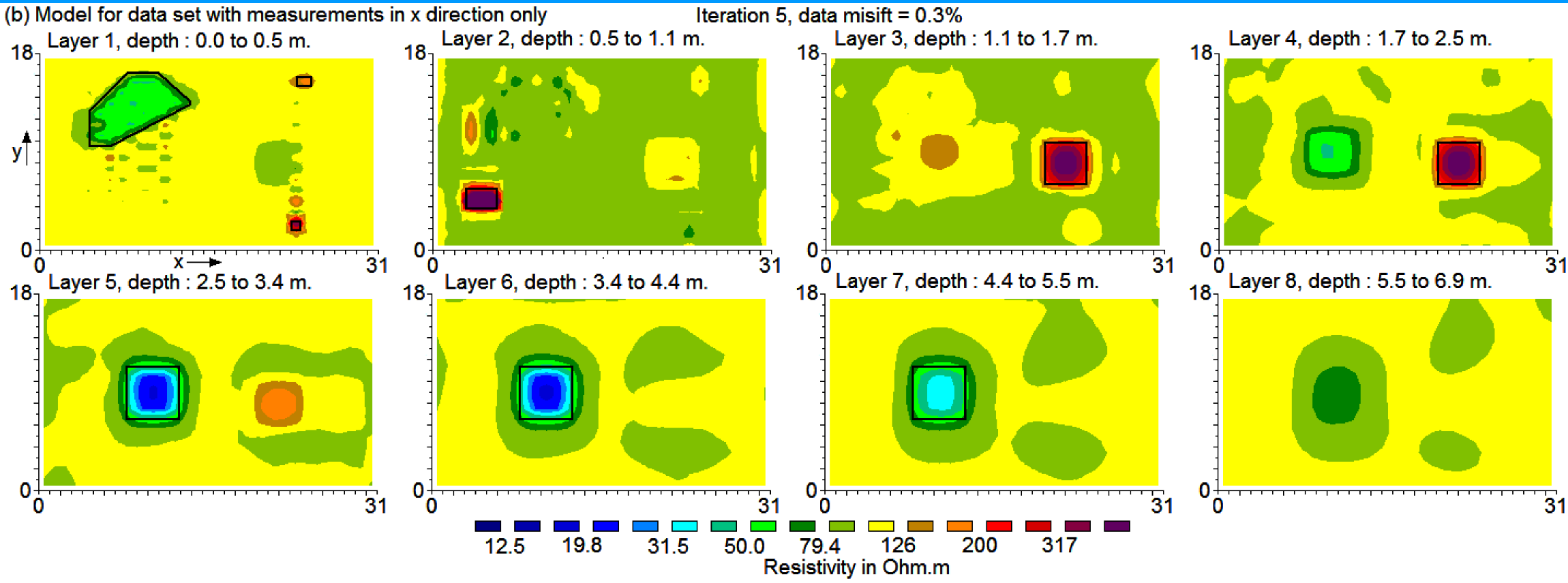
Test model – measurements in *x* and *y* directions

All possible the dipole-dipole measurements in both the *x* and *y* directions (but no diagonal measurements) with a geometric factor of less than 1056 m. (that corresponds to a dipole-dipole array with *a*=1 m. and *n*=6) are used. The main structures are fairly well resolved in the inversion model for this data set. The shape slanting boundaries of the near-surface low resistivity zone with slanting boundaries is accurately reproduced.



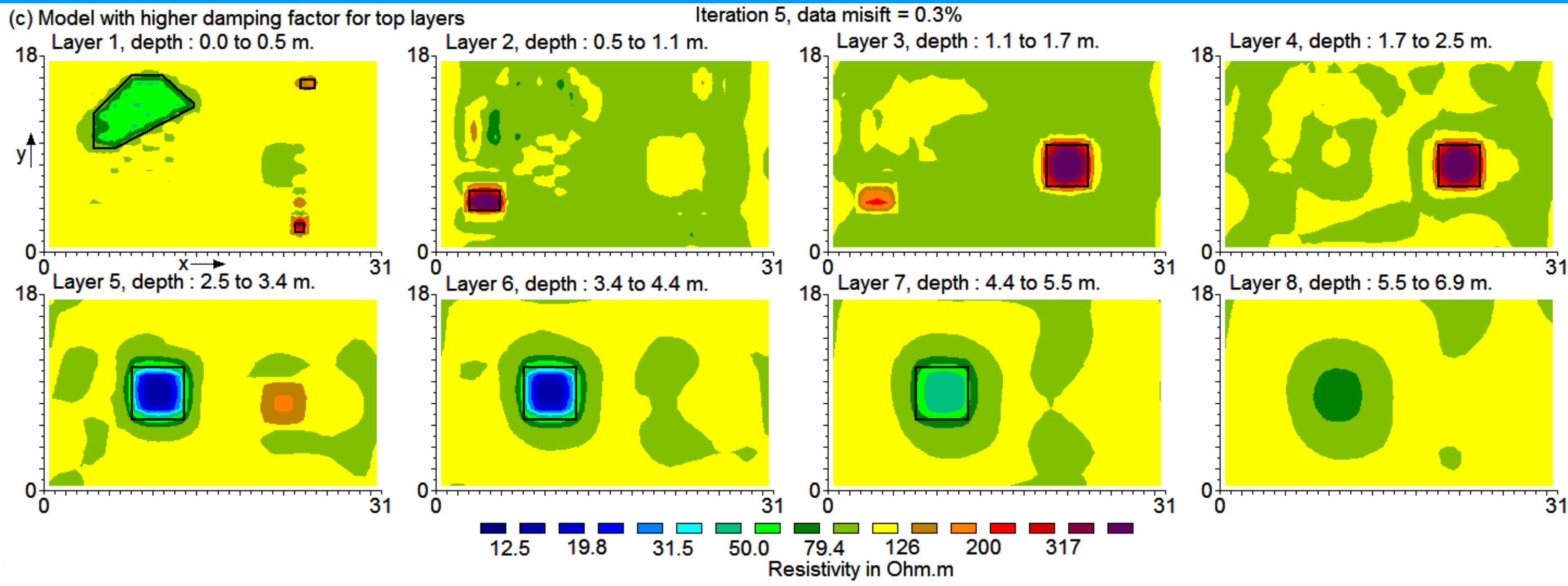
Test model – measurements in *x* direction only

Only the dipole-dipole measurements in the *x* direction are used. The model shows slight banding effects in the top layer with a notch at the leftmost boundary of the low resistivity region. There are slight elongated variations in the top layer near the low resistivity zone. The bottom of the deeper 500 Ω.m resistivity block is less well resolved with a trace of it in the 5th model layer that is not present in model with the *x* and *y* direction data.



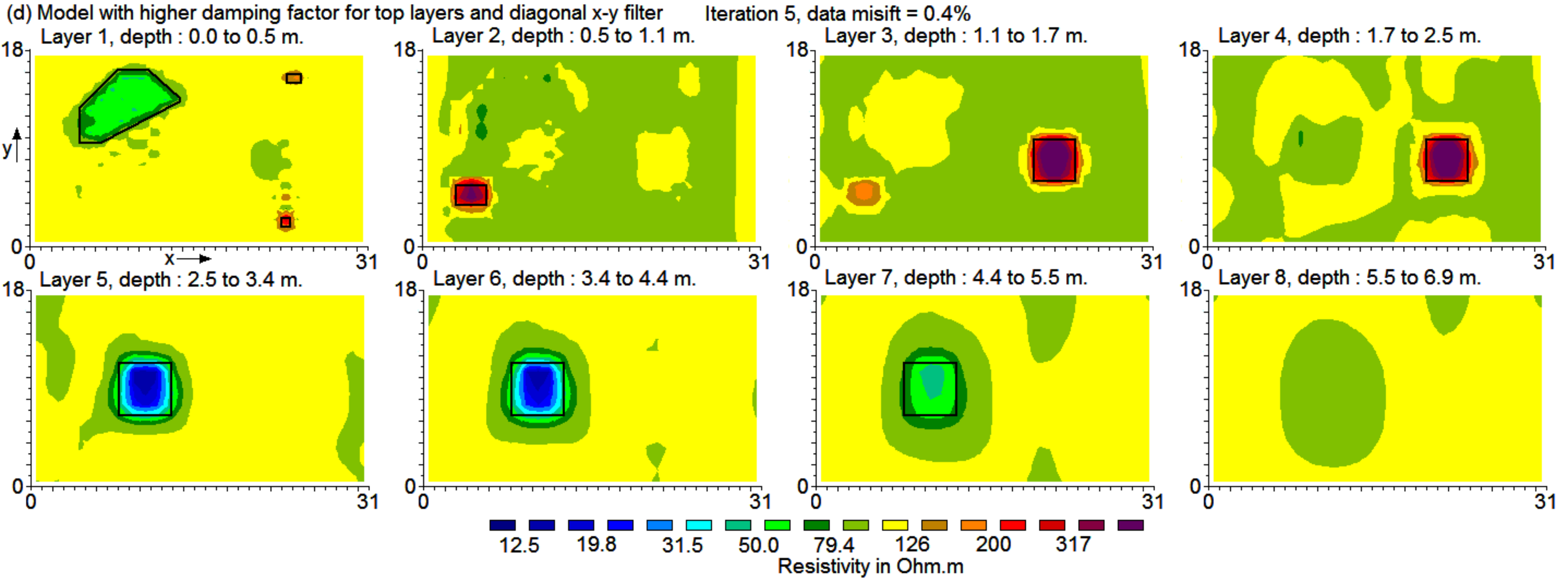
Model with higher first layer damping factor

Next, a higher damping factor of 3 times the normal value is applied to the top layer. This reduces the notch on the left boundary of the low resistivity zone and the 'ripples' near the surface. However the 'leakage' of the high resistivity anomaly from layer 4 to layer 5 is slightly higher.



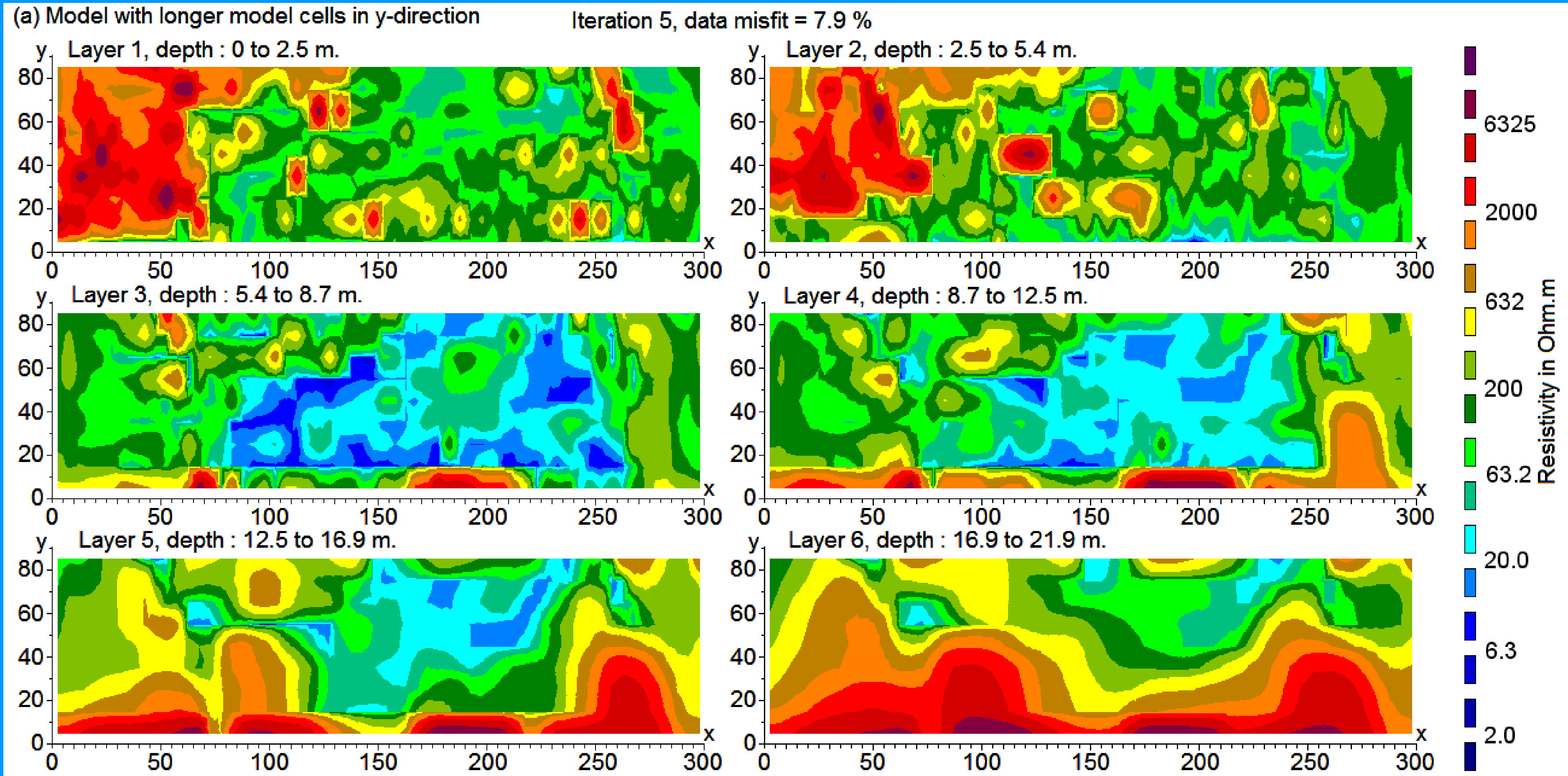
Model with diagonal *x-y* filter

We next modify the *x* and *y* roughness filters to include diagonal components in the *x-y* plane as well. The resulting model further reduces the banding effects in the top layer and the 'leakage' of the high resistivity anomaly into the 5th layer. The artefacts in layers 3 and 4 caused by the low resistivity block in layers 5 to 7 is also much reduced.



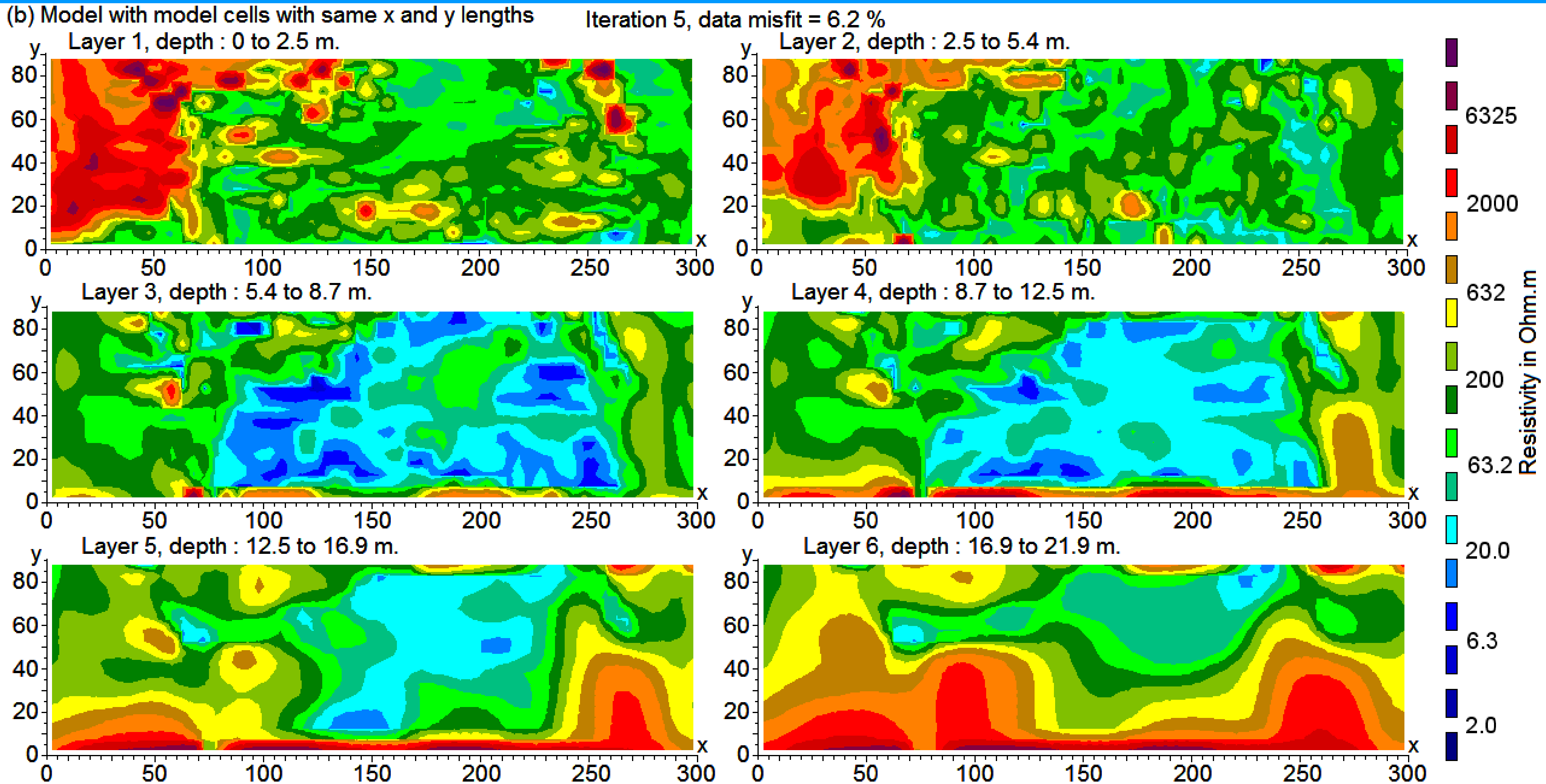
Ekeboda landfill data set – default model

The field survey data set consists of 10 parallel lines with 61 electrode positions along each line. The in-line electrode spacing is 5 meters, and the spacing between the lines is 10 meters. The model has prominent structures in the top two layers that are elongated in the *y* direction as the model cells are twice as long in this direction.



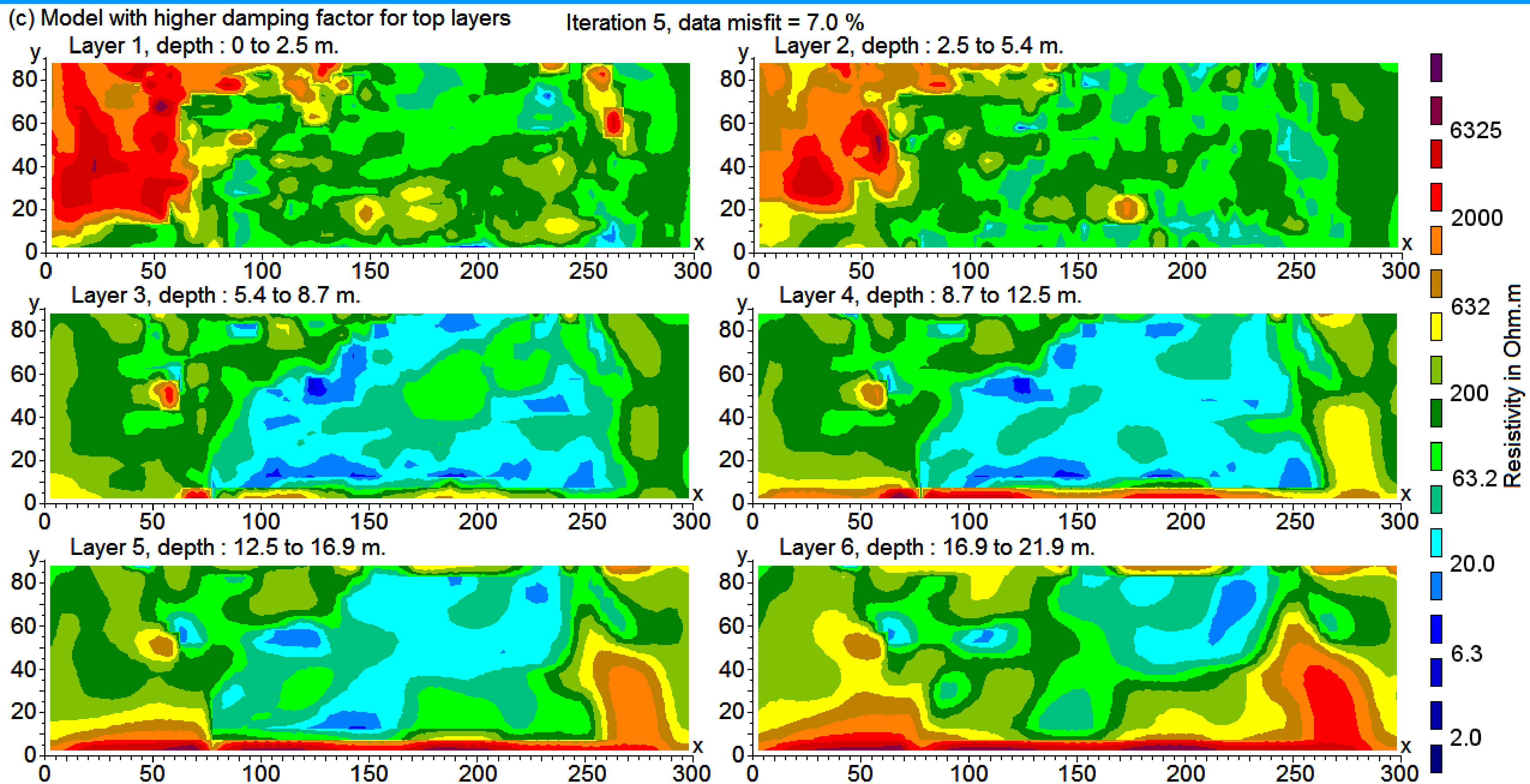
Ekeboda landfill – model with uniform cell lengths

The inversion model with cells of the same lengths in the *x* and *y* directions removes the elongated structures in the *y* direction. The banding effect in the *x* direction is more clearly shown in the top three layers, such as in the low resistivity (blue) landfill.



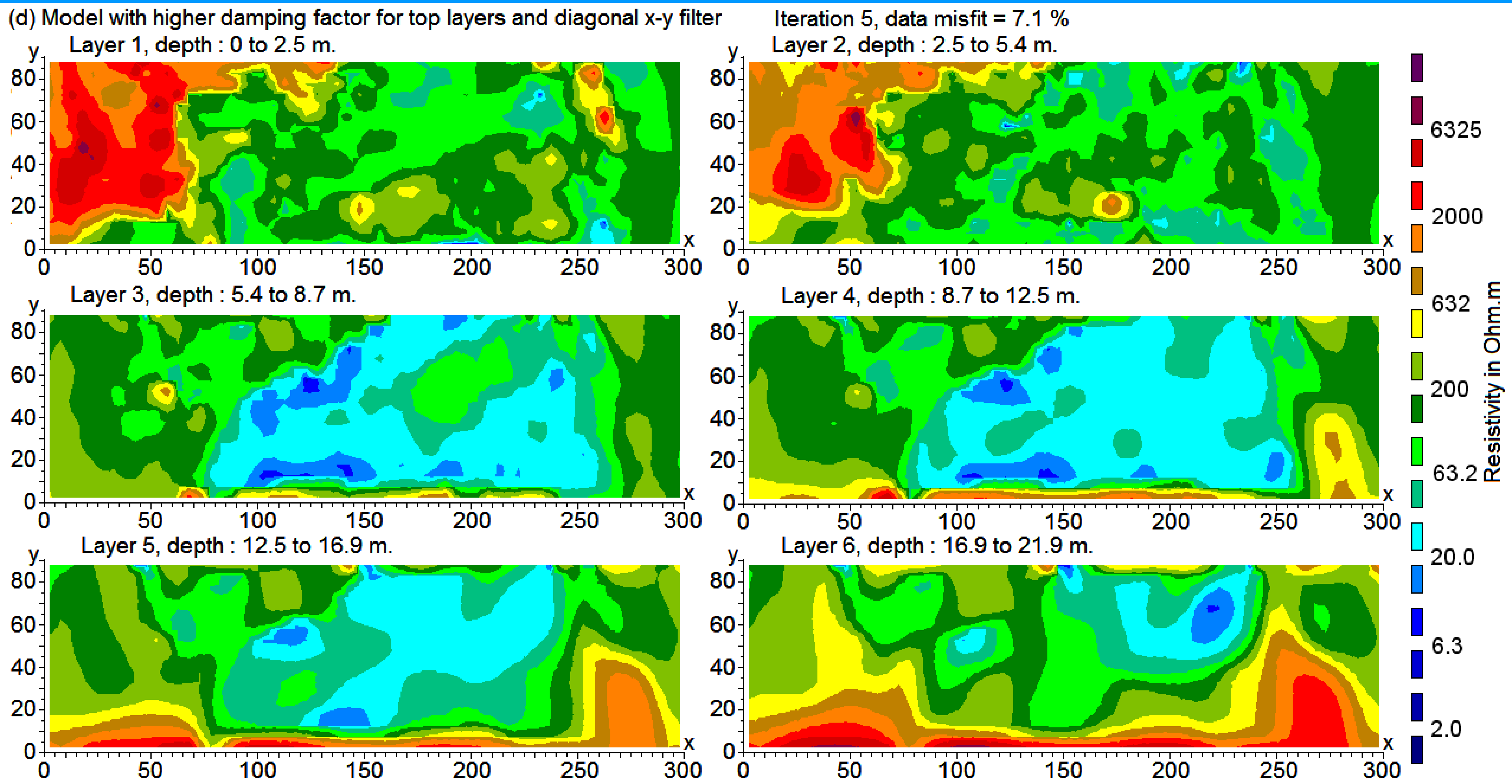
Ekeboda landfill – model with higher damping factors

The elongated structures in the *x* direction are greatly reduced by using a higher damping factor for the top layers. Note the more slanting left boundary of the low resistivity landfill.



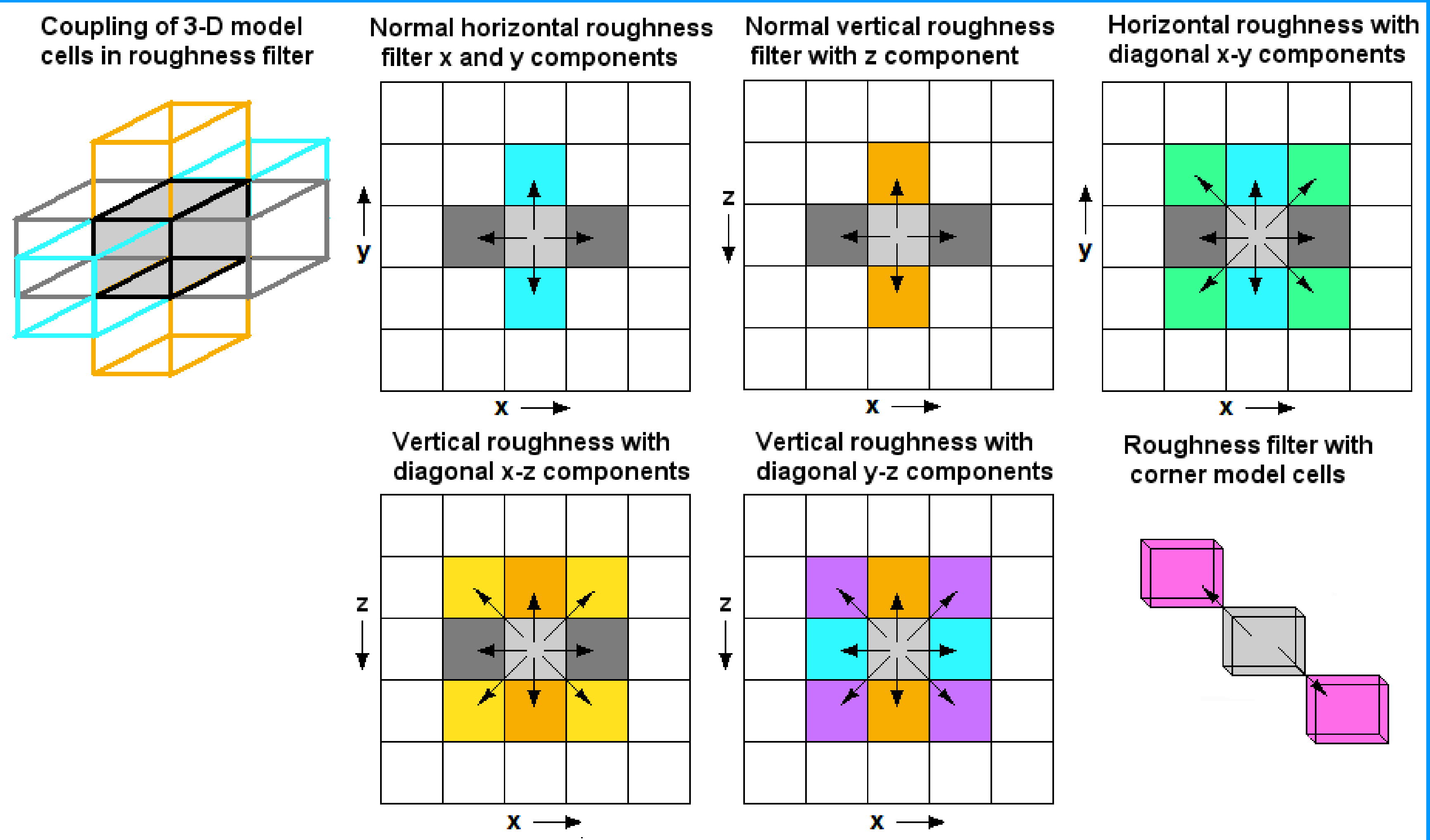
Ekeboda landfill data set – model with diagonal filters

The elongated structures in the *x* directionis are almost completely removed when the horizontal roughness filter with diagonal components is used.



Other types of diagonal filters

Roughness filters with diagonal components in the *x-z* and *y-z* directions can be used to reduce bias in the vertical direction. The roughness filter can also be applied between the central and corner cells as well (only 2 out of 8 corner cells are shown).



Conclusions

Many 3-D data sets are collated from a series of parallel 2-D survey lines. The distance between the lines is often two or more times the in-line electrode spacing.

Inversion models for such 3D data sets can display artefacts in the top layers elongated along the axes of the survey grid.

The artefacts are reduced by using a model discretization where the cells have about the same lengths in both horizontal directions.

Further reductions in the artefacts are achieved by using a higher damping factor for the top few layers together with a horizontal roughness filter with diagonal components in the least-squares optimization method used for the data inversion.